A Framework for Formal Reasoning about Geometrical Optics

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Abstract. Recently, optics technology has emerged as a promising solution by resolving critical bottlenecks in conventional electronic systems. Its application domain spans over diverse fields ranging from laser surgeries to space telescopes. In this paper, we describe an ongoing project which aims at building a theorem proving based framework for the formal reasoning about geometrical optics, an essential theory required in the design and analysis of optical systems. Mainly, we present the motivation of our work, a road-map to achieve our goals, current status of the project and future milestones.

1 Motivation and Background

Generally, optical systems are composed of different components (e.g., mirrors and lenses) which process light to achieve desired functionalities such as light amplification, ultrashort pulse generation and astronomical imaging. In order to model and analyze the behavior of such systems, light can be characterized at three levels of abstraction, i.e., ray, electromagnetic and quantum [4]. Geometrical optics (also known as ray optics) describes light as a collection of straight lines which linearly propagates through optical systems. On the other hand, electromagnetic and quantum optics characterize light as a coupled vector field and a stream of photons, respectively. The analysis of engineering optical systems (e.g., refractometry of cancer cells and optical networks) using geometrical optics is an integral part of their design life-cycle. Traditional optical system analysis techniques like paper-and-pencil based proofs and numerical algorithms have some known limitations of human-error proneness and incompleteness, respectively, which impeded their usage in the designing of critical optical systems which may result in the loss of human lives (e.g., laser surgeries) or heavy financial loss (e.g., Hubble Telescope failure [1]). We therefore propose theorem proving based formal methods for the accurate and scalable analysis of optical systems.

In this paper, we present details of an ongoing project[1] to develop a formal reasoning support for the analysis of geometrical optics. We use the HOL

[1] http://hvg.ece.concordia.ca/projects/optics/rayoptics.htm

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Light theorem prover to formalize the underlying theories of geometrical optics. The main reasons of our choice are the existence of rich multivariate analysis libraries as well as the active projects like Flyspeck [3]. This project is part of larger program on the formal analysis of different forms of optics (i.e., ray, wave, electromagnetic and quantum) [2].

2 Formal Analysis Framework

The proposed framework, given in Figure 1, outlines the main idea to formally model and prove that the optical systems model satisfies the system specification. The whole framework can be decomposed into four major parts which are

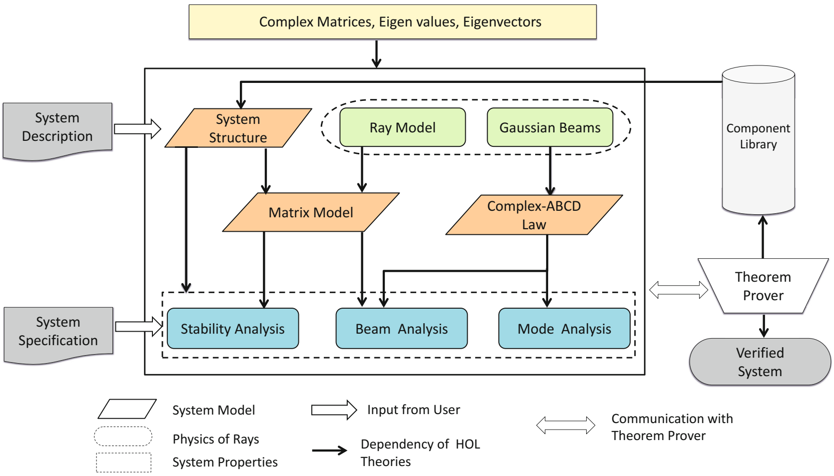

Fig. 1. Formal Analysis Framework

depicted by different shapes and colors as shown in Figure 1. First, the formalization of some complex linear algebra concepts such as complex matrices and eigenvalues; Second, the formalization of optical system structure; Third, modeling of rays and Gaussian beams and last, the formalization of the properties of optical systems such as stability, mode and output beam analysis. The two inputs to the framework are the description of the optical system and specification, i.e., the spatial organization of various components and their parameters (e.g., radius of curvature of mirrors and distance between the components etc.). The first step in conducting formal analysis is to construct a formal model of the given system in higher-order logic. In order to facilitate this step, we require a formalization of optical system structures which consist of definitions of optical interfaces (e.g., plane or spherical) and optical components (e.g., lenses and mirrors). The second step is to formalize the physical concepts of ray and

Gaussian beams. Building on these fundamentals, the next step is to derive the matrix model of the optical system which is basically a multiplication of the matrix models of individual optical components. This step also includes the formalization of the complex ABCD-Law of geometrical optics which describes the input-output relation of the given ray and Gaussian beams parameters.

Furthermore, in order to facilitate the modeling of system properties and reasoning about their satisfaction in the given system model, we provide their formal definitions and most frequently used theorems. These properties are *stability* which ensures the confinement of rays within the system, *beam analysis* which provides the basis to derive the suitable parameters of Gaussian beams for a given system structure and *mode analysis* which is necessary to evaluate the field distributions inside the optical system. Finally, we develop a library of frequently used optical components such as thin lenses, thick lenses and mirrors. Such a library greatly facilitates the formalization of new optical systems which are composed of these components as shown in Figure 1. The output of the proposed framework is the formal proof that certifies that the system implementation meets its specification. The verified systems will then also be available in the library for future use either independently or as part of a larger optical system. In practice, optical components are two dimensional and it is compulsory

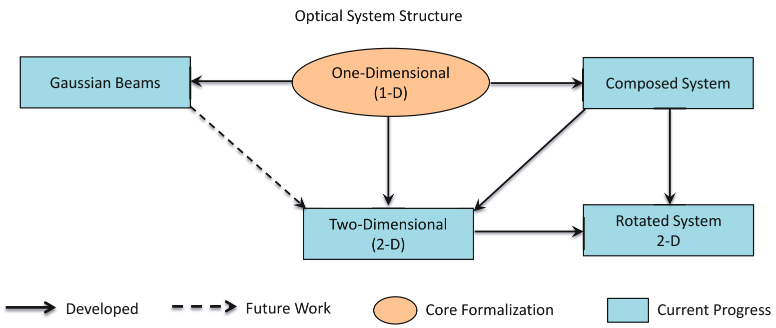

Fig. 2. Formalization Flow of Optical Systems Structure

to consider the rotational effects of individual components. Another important aspect is to consider the fact that optical systems are composed of small subsystems which are configured in a particular way to achieve desired functionalities. In our framework, we consider all of the above mentioned requirements in a systematic way as shown in Figure 2.

3 Current Status and Future Milestones

So far, we have developed a core formalization of geometrical optics [7] as shown in Figure 2. We aslo developed a library of frequently used optical components (such as thin lens, thick lens and dielectric plate) [7] and the formalization of optical resonators [6,7]. We showed the effectiveness of developed theories by the formal analysis of practical optical resonators like Fabry Pérot resonator with fiber-rod lens and Z-shaped resonators [6,7]. Moreover, we developed a generalized procedure for the formal stability analysis of optical resonators usable by physicists and optical engineers (details can be found in [5]). Recently, we have formalized two-dimensional and composed optical systems along with the formalization of Gaussian beams.

Finally, we outline the major tasks to achieve the remaining milestones until the end of this research project. According to our assessment, it would require additional 1.5 years (total project duration of 3.5 years) by an expert user of HOL Light with a sufficient background of geometrical optics. This time-line includes the extensions and revisions of existing formalization along with the dissemination of developed results. Following is the list of main tasks:

- Formalization of eigenray stability for periodic optical systems [8].
- Formalization of Gaussian beams in 2-D as shown in Figure 2.
- Formalization of misaligned optical systems.
- Extension of Gaussian beam formalization to handle laser mode-locking.

4 Conclusion

The main contribution of this project is a comprehensive framework of formal definitions and theorems ranging from one-dimensional ray optics to the rotated two-dimensional optical systems. Moreover, it can be considered as a one step towards an ultimate goal of applying formalized reasoning in new domains such as biology, physics and mechanical engineering.

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