Enabling the DC Solutions Characterization using a Fuzzy Approach

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***Abstract*—This paper presents a method for characterizing the DC operating points of analog circuits. We construct fuzzy DC equations that model circuit parameter variations and apply a global optimization algorithm to estimate the location of DC points. We applied our method to analyze the stability of a ring oscillator and the influence of the input voltage offset on the DC characteristic of a differential pair. Our results prove the effectiveness of our method in describing circuits DC performance parameters and predicting possibilities of undesired circuit operations.**

I. Introduction

As designs are moving into submicron technologies, analog effects are becoming a major concern during the verification at the circuit level. A first step in analog verification should start with a study of the main properties of the circuit, such as identifying its operating points. Such task is crucial because it directly affects the performance and yield of the circuit. In addition, this analysis is problematic, since finding equilibrium points consists in searching for the roots of a non linear vector valued function [1]. Moreover, the root search problem becomes more complicated under circuit parameters variation that directly affect the location and stability of DC operating points. For example, equilibrium points may move about or change their properties, as the parameters of the circuit vary.

Most efforts in formal methods for circuit steady state characterization have focused on using linear arithmetic solvers [2] [3] [4] where environment variations are modeled using a set of intervals. Besides the high computational time and the shortcomings of interval arithmetics, those approaches are often incomplete. The reason is the abstraction of transistor models by piecewise linear approximations that can easily miss operating points and fail to capture the real steady state behavior of the circuit. Monte Carlo methods [5] are typically used to manage device variation with steady state analysis. This method requires a large number of simulations to be evaluated. Also, due to the extremely large number of parameters affected by process variation and the complexity of analog circuits, such technique is usually time consuming and non exhaustive.

Global non linear optimization involves a set of techniques used to find the extrema of constrained nonlinear multivariable functions [6]. Fuzzy dynamical models are an alternative to introduce uncertainty in deterministic dynamical models [7]. Such models can be used for analog circuits to explore their state space and study the effect of their parameters variation [8]. In this work, process variation and circuit imperfections are incorporated as fuzziness in DC equations. Then, a global optimization algorithm is employed to determine bounds of the DC solutions and characterize the DC behavior deviation over these bounds.

In contrast to [2] [3] [4], our method uses fully non linear device models, integrates efficiently device uncertainty using fuzziness and provides a way to automatically compute the bounds of any circuit parameter performance such as stability, DC gain, noise margin and power using the obtained operating points characterization. We do not claim finding the solution to the problem of locating all the equilibrium points of analog circuit. However, our method is capable of providing useful information about the circuit reliability and robustness.

The rest of the paper is organized as follows: Section II details our methodology for DC operating points characterization using a fuzzy based modeling of the circuits and a global optimization technique. We also propose an algorithm for testing the effect of parameter variation on equilibrium points properties. In Section III, we provide experimental results for two analog circuits: a ring oscillator and a differential amplifier. In Section IV, we present our conclusions and future work.

II. DC Solutions Characterization Methodology

An overview of our proposed methodology for estimating the location of DC equilibrium points under parameter variations is shown in Figure 1. The DC equations are transformed into a set of non linear constraints. These non linear constraints and the fuzzy circuit parameters are input to a global non linear optimization step which estimates the DC solutions location. The accuracy of the fuzzy global optimization is enhanced by providing an initial guess determined during a previous DC sweep for nominal circuit parameters. The resulting fuzzy distribution can be analyzed in order to predict the circuit sensitivity to the parameters variation.

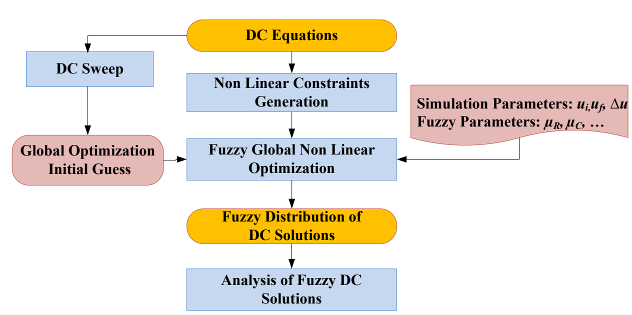

Figure 1. Fuzzy DC solutions characterization

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A. Fuzzy DC Equations

Analog circuit DC equations have usually the form:

$$\begin{aligned} \mathbf{f}(\mathbf{x}, \underline{\mathbf{u}}, \mathbf{P}) &= \mathbf{0} \\ \mathbf{P} &= \mu_{\mathbf{P}} \end{aligned} \quad (1)$$

where $\mathbf{f}$ is a non linear function, $\underline{\mathbf{u}} \in \mathbf{R}^m$ is a DC input and $\mathbf{P}$ is a set of circuit parameters related to the physical characteristics of the components. We model these parameters as fuzzy set [7] $\mu_{\mathbf{P}}$ defined using a membership function, as given in Figure 2.

Since it is not always possible to solve analytically the given set of DC equations for a specified fuzzy distribution, we need to convert it to a form that can be accepted by numerical solvers and still express the uncertainty of parameters. Similar to [9], we consider that a possibility distribution is a collection of $h_{\alpha cut}$ corresponding to a level h from 0 to 1, the resulting set of intervals can be processed iteratively. The $h_{\alpha cut}$ of a fuzzy set having a triangular distribution is an interval of the form $[(1-h)a + hb,\ hb + (1-h)c]$ where (a, b, c) are the parameters of the triangular membership function as shown in Figure 2. The $h_{\alpha cut}$ of a fuzzy set with a Gaussian distribution is defined as $[\mu - \sqrt{-2\sigma^2 \ln h},\ \mu + \sqrt{-2\sigma^2 \ln h}]$ where σ, μ are the parameters of the Gaussian membership function, as shown also in Figure 2.

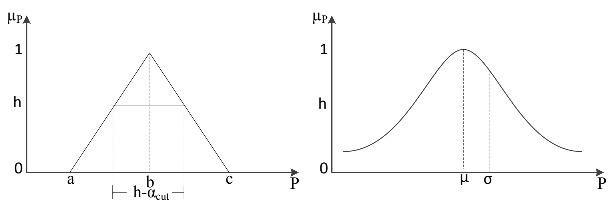

Figure 2. Fuzzy membership function examples

B. Non linear Constraints Generation

In the proposed methodology, the non linear constraints are:
- A number of non linear equalities, $C_{eq}(x, \underline{u}, P) = 0$ which are equivalent to the DC equations $f(x, \underline{u}, P) = 0$.
- A set of non linear inequalities, $C_{ineq}(h_{\alpha cut}, P) \leq 0$, which express that each parameter lies in the $h_{\alpha cut}$ interval.

C. Fuzzy Global Non linear Optimization

Solving the fuzzy DC equations is transformed into a multi-variable optimization problem, where the circuit parameters are constrained to a continuum set of $h_{\alpha cut}$. Algorithm 1 provides a description of the fuzzy global optimization procedure to solve the problem given in Equation (2).

$$min, max\ \mathbf{x}\ subject\ to \left\{ \begin{array}{lcl} C_{ineq}\,(h_{\alpha cut}, P) & \leq & 0 \\ C_{eq}\,(\mathbf{x}, \underline{u}, P) & = & 0 \end{array} \right. \quad (2)$$

The requirements of Algorithm 1 are:
- A set of constraints $mycons$ generated in the previous steps.
- The input range parameters: starting, final and sweep values of the circuit source $(u_0, u_f, \Delta u)$.
- The initial guess $\mathbf{x_0}$ of the global optimization step, determined by a DC sweep.

Alg. 1 Fuzzy Global Non Linear Optimization

Input: $u_0, u_f, \Delta u, dist, \mathbf{x_0}, \Delta h, h_0, n, mycons, alg$
Output: $\mathbf{x}_{min}, \mathbf{x}_{max}$

1: **for** $h = h_0 + \Delta h\ to\ 1$ **do**
2: **for** $u = u_0 + \Delta u\ to\ u_f$ **do**
3: **for** $i \in 1\ to\ n$ **do**
4: $[\mathbf{x}_{imin}, \mathbf{x}_{imax}] = GO(x_i(u, \mathbf{x_0}), alg, mycons)$
5: **end for**
6: **end for**
7: **end for**

- The type of the algorithm to use during the optimization which is the interior point [10] in our case.
- The step value Δh level that depends on the desired degree of accuracy.

For each level h from h_0 to 1, a constant input u and an initial guess $\mathbf{x_0}$ computed via DC sweep, a Global Optimization (GO) procedure determines the minimum and the maximum of the equilibrium point for that fixed input u when the circuit parameters vary in the $h_{\alpha cut}$ interval (line 4). The superposition of the computed intervals $[\mathbf{x}_{imin}, \mathbf{x}_{imax}]$ at each iteration forms a fuzzy distribution of the circuit DC equilibrium points.

D. Analysis of Fuzzy DC Solutions

The last step of the methodology is the analysis of the obtained fuzzy DC sets which characterize different circuit performances. To illustrate this step, we focus on how to perform circuit stability analysis [11]. Equation (3) provides the stability conditions of an equilibrium point $\mathbf{x}$ based on the sign of the real part of the eigenvalues of the Jacobian matrix J of the DC equations at $\mathbf{x}$ and for a DC input $\underline{u}$.

$$\begin{aligned} \mathbf{J} &= \frac{\partial \mathbf{f}}{\partial \mathbf{x}}_{|x, \underline{u}} \\ \mathbf{\Omega} &= \mathbf{eig}(\mathbf{J}) \\ \textbf{if}\ \forall\ \omega \in \mathbf{\Omega},\ \mathbf{Real}(\omega) < \mathbf{0} &\Rightarrow \textbf{stable x} \\ \textbf{if}\ \exists\ \omega \in \mathbf{\Omega},\ \mathbf{Real}(\omega) > \mathbf{0} &\Rightarrow \textbf{instable x} \end{aligned} \quad (3)$$

Algorithm 2 shows our approach to analyze the stability of the given circuit over the range of the fuzzy DC states generated in the previous step. Therefore, in order to claim that a circuit is stable for a fuzzy DC state, we need to show that for each $h_{\alpha cut}$ all the maximum of Λ, defined in line 3, is negative. This concludes that at any point of the fuzzy DC description, all the jacobian eigenvalues real parts are negative. Alternatively, if for each $h_{\alpha cut}$, the maximum of Λ is positive, we can conclude that the fuzzy DC point includes at least one instable operating point. However, if we succeed to prove for each $h_{\alpha cut}$ that the minimum of Λ is also positive, we confirm that the circuit is always instable and never locks to a fixed state with the parameter values used.

III. Applications

In this section, we apply our method to estimate DC operating points locations and analyze their stability for a Three-Stage Ring Oscillator subject to a fuzzy description related to the size of its inverters. Also, we examine the effect of a fuzzy input voltage offset on the DC operating points and the performance of a Differential Amplifier. All simulations were conducted in a MATLAB environment [12].

Alg. 2 Stability Analysis

Input: $u_0, u_f, \Delta u, \Delta h, h_0, alg$
Output: max, min
for $h = h_0 + \Delta h$ *to* 1 **do**
  for $u = u_0 + \Delta u$ *to* u_f **do**
    $\Lambda := sup(real(eig(J(x, u)))$
    $mycons := x \in [x_{min}(h), x_{max}(h)]$
    $x_0 = (x_{max}(h) - x_{min}(h))/2$
    $[min_\Lambda, max_\Lambda] = GO(\Lambda, x_0, alg, mycons)$
  end for
end for

A. Ring Oscillator

Figure 3 represents a ring oscillator built from three inverters connected in series. Each inverter is composed of a cascaded nmos and pmos transistors.

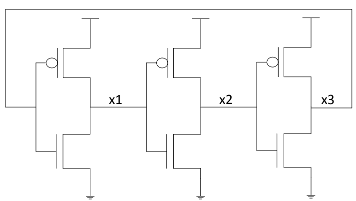

Figure 3. 3-Stage Ring Oscillator

The inverters transistors width ratio $r = \frac{w(pmos)}{w(nmos)}$ affects the location and the stability of the circuit DC equilibria [13]. We apply our methodology to answer the questions: *how does the variation of the inverters size affect the operating points location?* and *is there any possibility that the circuit locks?* Therefore, we model the parameter r as a fuzzy set which leads to Equation (4), where the first three lines and the last two lines correspond to the non linear equalities C_{eq} and inequalities C_{ineq} constraints, respectively.

$$\begin{aligned} I_n(x_3, x_1, gnd, r) + I_p(x_3, x_1, vdd, r) &= 0 \\ I_n(x_2, x_1, gnd, r) + I_p(x_2, x_1, vdd, r) &= 0 \\ I_n(x_2, x_3, gnd, r) + I_p(x_2, x_3, vdd, r) &= 0 \\ r - h_{\alpha cut max} &\leq 0 \\ h_{\alpha cut min} - r &\leq 0 \end{aligned} \quad (4)$$

where x_1, x_2 and x_3 are the voltages at the output of each inverter, $gnd = 0$ is the ground voltage and $vdd = 1.8V$ is the power supply voltage. The functions I_n and I_p model the non linear current generated by the nmos and pmos transistors, respectively, based on their gate, drain and source voltages. The objective function during the GO procedure is the circuit state variable vector. At each iteration, the parameter r belongs to an $h_{\alpha cut}$. Table I summarizes the results from the analysis of the ring oscillator DC solutions. In the first experiment, we define the fuzzy number μ_r using a triangular membership function of parameters (a=1, b=2, c=3).

Figure 4(b) shows the obtained results. For each interval $h_{\alpha cut}$, in which the parameter r lies, we locate a single region $[x_{imin}, x_{imax}]$ of equilibrium points. The minima are represented by the red color while the maxima are in blue. The superposition of the located regions forms a possibilistic triangular distribution offering an overapproximation that covers all the possible values of the operating points and maintains the same membership function of the parameter r. If we suppose

Table I. EXPERIMENTAL RESULTS

	Experiment 1	Experiment 2
Membership function	triangular	gaussian
Fuzzy parameters	a=1, b=2, c=3	$\sigma = 0.1, \mu = 0.4$
Performance results	Stable oscillation ✓	May oscillate or not **X**

that the circuit is initially designed to work properly such as the width of the pmos transistor is twice the nmos one, the voltage at each output node of the inverters is in this case $0.85V$. In the worst case scenario in the proposed triangular distribution of the r parameter, the operating point may reach a minimum of $0.76V$ which is approximately a degradation of 10% or a maximum of $0.9V$ which is an increase of approximately 5%. Those information can be very useful to the designer to predict the performance parameters sensitivity over transistor size variation. We analyze the stability of the DC

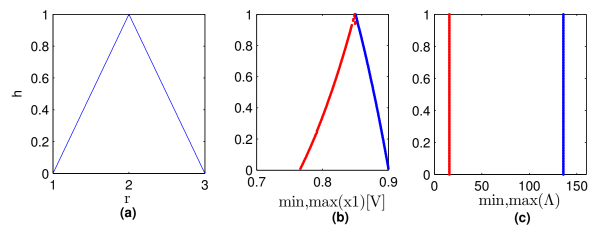

Figure 4. Fuzzy triangular distribution and stability testing

points distribution as described in Algorithm 2. The maximum and minimum of Λ when the DC points belong to each located interval $[x_{imin}, x_{imax}]$ are also shown in Figure 4(c). For all $h_{\alpha cut}$, the minimum of Λ is always positive, the DC equilibria in the located possibilistic distribution are then all unstable. We notice that the variation of the parameter r as considered in this example does not affect the properties of the DC equilibria (unstable). Similar results were found for other odd stage ring oscillators. Using our proposed approach, we can conclude that an odd stage ring oscillator will not lock at a stable DC equilibrium point. The ring oscillator will probably maintain stable oscillations. Same results were found in [14] but for a fixed value of the inverter size $r = 2$, while in our experiment we assume that we have qualitative, uncertain knowledge about the value that enables us to cover different circuit realizations. In the second experiment, we propose

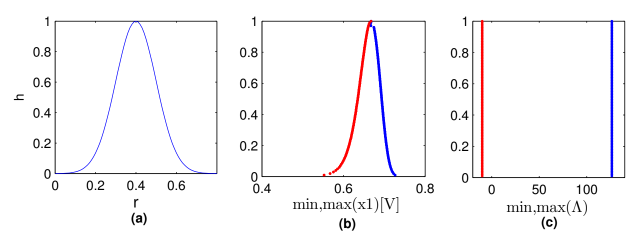

Figure 5. Fuzzy Gaussian distribution and stability testing

to estimate the location of DC equilibrium points when the parameter r is very small which means that the width of the nmos transistor is much larger than the pmos one. We define the fuzzy number μ_r using a Gaussian membership function of parameters ($\sigma = 0.1, \mu = 0.4$). The resulted region forms a possibilistic Gaussian distribution as shown in Figure 5(b). The experimental results are summarized in Table I.

The stability testing results in Figure 5(c) shows that the minimum of Λ is always negative which means that there exist stable equilibrium points in the located distribution. Therefore, the variation of the parameter r may change the property of the DC equilibrium point from unstable to stable. It is then possible that the operating point settles at some stationary state and causes a failure to oscillation.

B. Differential Amplifier

We consider a CMOS differential amplifier [13] that consists of a current mirror and a differential pair as shown in Figure 6. The performance of such circuit is very sensitive to device mismatch. Such mismatch leads to a random offset that we model as a fuzzy input offset voltage inserted at one of the input terminal, leading to Equation (5). The objective of our experiment is to analyze the effect of the offset on the differential pair operating points and characterize its performance.

$$\begin{aligned} I_n(V_{os}, x_1, x_3) + I_p(x_1, x_1, vdd) &= 0 \quad (5)\\ I_n(vss, x_2, x_3) + I_p(x_1, x_2, vdd) &= 0\\ I_n(V_{os}, x_1, x_3) + I_n(vss, x_2, x_3) - I_n(x_4, x_3, vss) &= 0\\ V_{os} - h_{\alpha cut max} &\leq 0\\ h_{\alpha cut min} - V_{os} &\leq 0 \end{aligned}$$

In Figure 7(a), we consider the offset voltage Gaussian distribution where $\sigma = 0.1$ and $\mu = 0$, to model the mismatch. The minimum and the maximum of the voltage at the output, given in Figure 7(b), form a Gaussian distribution that covers all the possible equilibria locations and offers a complete estimation of the equilibrium values. Ideally, if we ground both inputs, the output voltage is zero. The circuit is desired to work around that operating point to ensure a maximum gain and a proper operation. Our results show that if the offset voltage magnitude is $0.2V$, then the output will be driven away from its ideal value and saturates either near a negative or positive voltage level ($\pm 2V$). Such output saturation causes a degradation of the amplifier performances. The effect of V_{os} on the stability

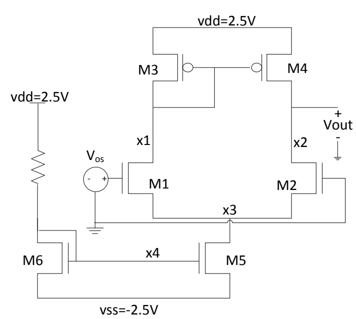

Figure 6. CMOS differential amplifier

of the DC operating points is analyzed using Algorithm 2. Ideally, when the input is $0V$, the output should remain stable. This scenario is illustrated in Figure 7(c) when the offset is $0V$ and the maximum of Λ is negative. The stability analysis shows that the minimum of Λ is always negative while the maximum becomes rapidly positive as the offset voltage magnitude increases. Due to the offset voltage, our approach shows how the operating point may easily become unstable and lead to an undesired behavior.

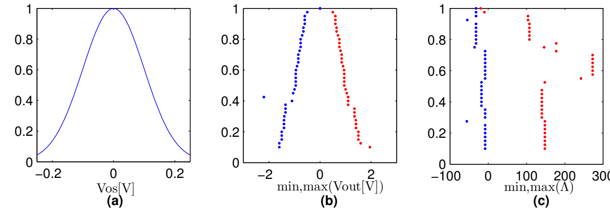

Figure 7. Gaussian distribution of equilibrium points and stability testing

IV. Conclusion

This paper proposed a methodology based on global optimization and fuzzy theory to characterize the sensitivity of DC operating points to parameters variation. The good estimation of the possible DC values allowed by the fuzzy modeling approach of analog effects offers a valuable means to predict the circuit behavior. Our approach is able to cover anomalous behavior of DC points that can cause circuits failure. It offers a better coverage than Monte Carlo method and avoids the oversimplification of circuit models and the huge running time of formal techniques for DC analysis. The accuracy of the outcome is comparable to that of a common method of optimization. The initial point can have a large effect on the solution. Evolutionary optimization algorithms, which use a population approach to increase the chance to converge to a global minimum, can address this limitation.

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