Discriminating Chaos from Non-Gaussian Noise on Analog Circuits

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***Abstract*—This paper presents an approach for studying nonlinear circuits dynamics. In particular, we study deterministic vs. stochastic dynamics by distinguishing chaotic behavior from noise. The proposed approach is general enough to detect any type of noise with no restriction on the noise distribution. It is based on a statistical proof by contradiction using hypothesis testing with Lempel-Ziv Complexity (LZC) method as discriminating statistics. The LZC method is adopted as a nonparametric test that has the advantage of detecting any kind of noise. Experimental results on a $\Sigma-\Delta$ Modulator and a Lorentz circuit showed that the proposed approach provides a reliable distinction between chaotic/hyperchaotic behavior and non-Gaussian noise.**

I. Introduction

Chaos refers to the unpredictable noise similar behavior that is difficult to be deciphered with an exponential sensitive dependency on initial conditions and circuit parameters. In fact, even very small initial condition variations exponentially change the behavior of the circuit over time which is known as the *Butterfly effect* [1]. Chaotic circuits present thereby very promising features that make them used in many applications such as communication [2], signal processing [3], and neural networks [4]. Nonetheless, the absence of a precise mathematical definition of the chaos phenomena makes it very challenging to be distinguished from noisy circuit behavior. Because noise is omnipresent in analog circuits, chaos detection is posing a real challenge to both designers and verification engineers due to its random like behavior. This is particularly important for applications wherein chaotic behavior is undesired and can cause catastrophic failures. Examples of this are deadly cardiac arrhythmias, a fatal voltage collapse in power networks, and regulating responses of machines [5]. It is therefore imperative to develop techniques and tools that can reliably discriminate noise from chaos in analog circuits.

For long decades, classical Lyapunov theory has been used as a hallmark of chaos because it was believed that chaotic circuits have unstable equilibrium points. A circuit is considered chaotic if it presents one or more positive Lyapunov exponent. Whereas, recent research showed that there exist chaotic circuits with stable equilibrium points [6]. Hence, a positive Lyapunov exponent is neither necessary nor sufficient proof of chaos. Simulation is the standard technique for analog circuits verification. However, available technology techniques for circuit verification, like spectral analysis, fail to discriminate chaotic from noisy circuit responses since both of them have continuous broadband power spectra [7]. Periodic Steady State (PSS) analysis was also used in [8] to discriminate periodic from chaotic behavior. A periodic behavior is detected when the obtained convergence norm is equal or less than unity. Conversely, a chaotic behavior is reported when the Spectre simulator does not converge and the PSS analysis fails to find any periodicity in the circuit response. Nevertheless, a non-periodic behavior could also emerge from noise and not from chaos. In addition, the Newton algorithm used by the PSS method requires a pre-known oscillation frequency as well as the computation of the Jacobian matrix of the output. Therefore, this technique suffers from serious scalability and applicability issues. A surrogate generation method is proposed in [9] to statistically probe chaos from noise. It is based on a surrogate generation method and hypothesis testing with Gaussian Kernel test statistic. However, this Gaussian Kernel test is suitable only for the case of Gaussian noise. Hence, a clear differentiation between chaotic and non-Gaussian noisy behaviors seem to be rather problematic.

In this paper, we are concerned with the verification of possible aberrant circuit dynamics that can be emerging from chaos due to process variations in the circuit parameters steming from fabrication imperfections. To do so, we extend the work proposed in [9] to handle noise that does not follow the shape of Gaussian distribution such as $1/f$ noise (a.k.a. flicker noise). The rest of the paper is organized as follows: Section II provides an overview of the proposed methodology. Experimental results of the verification of a $\Sigma-\Delta$ Modulator and a Lorentz circuit are reported in Section III. Section IV summarizes the contributions of this paper.

II. Proposed Methodology

Figure 1 details our surrogate based methodology to statistically discriminate chaos from noisy responses in analog circuits. Given a circuit topology, a design specification and a technology library, the analog circuit behavior is modeled as Extended-System of Recurrence Equations (E-SREs) [10] that describe its behavior with and without noise. The methodology starts by conducting transient simulations of the circuit behavior in light of process variations. Then, the surrogate verification scheme is deployed in the state-space domain using a statistical proof by contradiction hypothesis testing procedure. In the sequel, we detail the different steps and methods of the proposed methodology.

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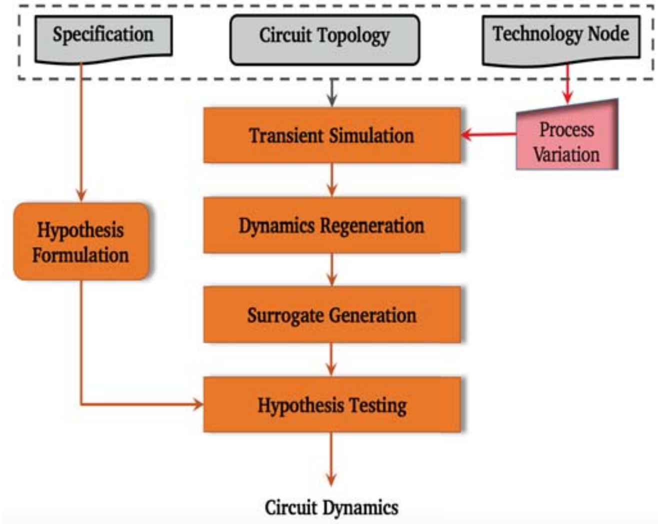

Fig. 1. Proposed chaos detection methodology

Extended-SREs (E-SRE) are extentions of differential equations describing the analog circuit behavior by expressing them with *if-else* logical formulas as follows:

$$X_i(n) = f_i(X_j(n-\gamma)), \forall i, j, n \in \mathbb{Z} \quad (1)$$

where $X_i(n) \in \mathbb{R}$ depicts the state variable and $\gamma \in \mathbb{N}$ denotes the time delay. f_i refers to a generalized If-formula $If(P, x, y) : B \times \mathbb{K} \times \mathbb{K}$ that satisfies the following two axioms:

$$\text{If } (true,\ x,\ y) = x$$
$$\text{If } (false,\ x,\ y) = y$$

In the proposed methodology, we first perform transient simulation of the obtained E-SRE analog circuit model for specific environment constraints, namely the initial values of the voltage and current state variables and the simulation parameters (such as the total simulation time and the simulation step size). Thereafter, a dynamics regeneration method is adopted for a *state-space* verification of the circuit dynamics. In the state-space domain, the circuit state variables are displayed against each other, i.e., it leaves time as an implicit dimension not explicitly graphed. The subset of this state-space domain towards which the circuit tends to evolve regardless of the initial conditions is called an *attractor*. This attractor $\mathcal{A}$ is used to predict the chaotic behavior of an Analog and Mixed Signal (AMS) circuit in order to consider them in the surrogates generation later. The non-uniform embedded window [11] and the false nearest neighbor method [12] are used to establish optimal embedding parameters (d_e, τ) for the attractor reconstruction. A surrogate generation method, originally proposed in [9], is carried out to generate artificial circuit outputs called *surrogates*. They are extracted from the circuit output so they are free from any chaotic process (i.e., noisy) while preserving some features of the circuit output. To do so, it constructs another circuit attractor $\mathcal{A}_n$ that exhibits noisy trajectories. The method selects an initial condition randomly from the reconstructed attractor $\mathcal{A}$. It then repeatedly chooses new points for the noisy attractor $\mathcal{A}_n$ with a probability commensurate with a certain noise radius ρ from a near neighbor $z_j \in \mathcal{A}$. Next, we elucidate the property of interest ($\mathcal{P}$) that the circuit should comply with. The property to be verified is phrased as follows: "*Is the observed random like behavior of the circuit emerging from noise or chaos?*". Hence, we define a null hypothesis, denoted by H_0, which assumes that the circuit exhibits stochastic noise and an alternative hypothesis H_1 that assumes the circuits to be purely deterministic, i.e., chaotic. To verify the above-mentioned hypotheses, the generated surrogates are considered as the null model against which the real circuit output is verified. Hence, chaotic and stochastic circuit dynamics lead to distinct trends of their generated surrogates produced while being verified.

Lempel-Ziv Complexity Test Statistic

The Lempel-Ziv Complexity (LZC) method, first defined in [13], is a nonparametric measure of complexity in the sense of Kolmogorov. It is able to capture randomness, i.e., the degree of redundancy (or patterns) that are similar in a signal without making any assumption about its distribution. Unlike the Gaussian Kernel Algorithm (GKA) proposed in [9], this measure has the advantage of handling stochastic circuit behavior that does not follow a Gaussian distribution. It objectively and quantitatively estimates system complexity through the change process of inherent system structure.
Consider a signal $X = (x_1, x_2, ..., x_N)$ of length N that takes its values in an alphabet A of finite size $\alpha = |A|$:

$$S_i = \begin{cases} 0 & \text{if} \quad x_i < T_d \\ 1 & \text{if} \quad x_i \geq T_d \end{cases}$$

The upper limit of the complexity counter is given by:

$$c(N) < \frac{N}{(1-\epsilon_N)\ log_\alpha(N)} \quad (2)$$

where α is the number of alphabets in the circuit output under verification (it is independent of the length of the output under verification N) and ϵ_N is given by the following equation:

$$\epsilon_N = 2\frac{1 + log_\alpha(log_\alpha(\alpha N))}{log_\alpha(N)} \quad (3)$$

The normalized LZC C_{LZ} is defined as follows:

$$C_{LZ}(N) = \frac{c(N)}{b(N)} \quad (4)$$

where $b(N)$ is given by the following equation:

$$b(N) = \frac{N}{log_\alpha(N)} \quad (5)$$

Ziv [14] proved that if x is the infinite length output from an ergodic source with entropy rate *h*, then

$$\limsup_{n \longrightarrow \infty} C_{LZ}(n) = h \quad (6)$$

III. APPLICATIONS

In this section, we report the results of the application of our methodology on a $\Sigma-\Delta$ modulator and a Lorentz circuit. All computation and circuit models were performed in a MATLAB environment and were run on a 64-bit Windows 7 server with 2.8 GHz processor and 24 GB memory. The type of hypothesis testing used is the one-tailed test with the level of significance $\alpha = 5\%$.

A. First-order $\Sigma - \Delta$ Modulator

The circuit diagram of a first-order $\Sigma - \Delta$ modulator is shown in Figure 2. It consists of a negative feedback loop with 1-bit quantizer and a discrete time integrator.

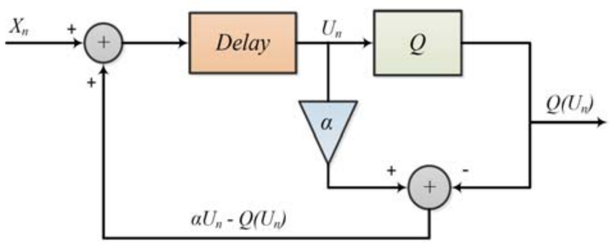

Fig. 2. First-order $\Sigma - \Delta$ modulator

The behavior of the circuit can be described by the following E-SRE:

$$U_n = \text{if } (U_{n-1} \geq 0, \alpha U_{n-1} + X_{n-1} - 1, \alpha U_{n-1} + X_{n-1} + 1))$$

where X_n and α stand for the input signal and the modulator gain, respectively. The nonlinearity being introduced by the quantizer block Q gives rise to a chaotic behavior if the gain α is in the range]1, 2]. However, if $\alpha \leq 1$, instead of chaos, the circuit exhibits a normal operation (i.e., the quantized output will be approximately equal to the input signal) [15]. We performed the verification of this circuit under two cases: (1) Chaotic regime with $\alpha = 1.14$; (2) Noisy regime by introducing non Gaussian flicker noise to a non-chaotic output for $\alpha = 0.5$. Figure 3 illustrates both behaviors as a function of time for a 1.5 KHz sine wave input. The top panel (a) represents the quantized output for a gain $\alpha = 0.5$ (ideal regime). The panel (b) represents the quantized output of the same input signal for an $\alpha = 1.14$ (chaotic regime). The third panel (c) of the same figure shows the noisy regime for the same input. It can be remarked that a noisy quantized signal with a non chaotic gain (Figure 3(b)) reveals a similar behavior to the chaotic circuit (Figure 3(c)). The results of the application of our verification methodology to the first-order $\Sigma - \Delta$ modulator are recapitulated in Table I. In the chaotic regime, both the Gaussian Kernel Algorithm (GKA) [9] and LZC methods rejected the null hypothesis H_0 of noisy circuit behavior. Nevertheless, only the proposed LZC method successfully discriminated the noisy behavior emanating from flicker noise. Indeed, the null hypothesis H_0 is found consistent with the noisy assumption whereas GKA test statistic falls short to do so. Consequently, our methodology

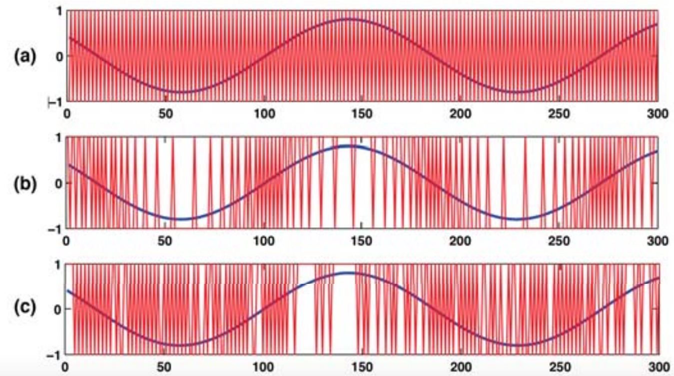

Fig. 3. The input sine wave and the quantized output of the $\Sigma - \Delta$ modulator in different regimes

TABLE I
RESULTS OF VERIFYING $\Sigma - \Delta$ MODULATOR

	GKA method [9]	Proposed LZC test
Chaotic Regime	Reject H_0	Reject H_0
Noisy Regime	Reject H_0	Accept H_0

outperforms the method proposed in [9] in detecting non Gaussian noisy behavior.

B. Lorentz based Circuit

The circuit depicted in Figure 4 reproduces the Lorentz system of equations. It is composed of six operational amplifiers, and four AD633 multipliers which implement the nonlinearity of the circuit [16]. The Lorentz circuit is governed by the following E-SREs:

$$\begin{aligned}
X(n+1) &= \text{if } (true, X(n) + \delta_t k(\frac{1}{R_2C_1}Y(n) - \frac{1}{R_1C_1}X(n) \\
&\quad + \frac{1}{10R_3C_1}Y(n)Z(n)), 0) \\
Y(n+1) &= \text{if } (true, Y(n) + \delta_t k\frac{R_{13}}{R_{12}}(\frac{1}{R_5C_2}X(n) - \frac{1}{10R_6C_2} \\
&\quad X(n)Z(n) - \frac{1}{R_4C_2}Y(n) - \frac{R_{15}}{R_{14}R_7C_2}W(n)), 1) \\
Z(n+1) &= \text{if } (true, Z(n) + \delta_t k(\frac{1}{10R_9C_3}X(n)Y(n) \\
&\quad - \frac{1}{R_8C_3}Z(n)), 1) \\
W(n+1) &= \text{if } (true, W(n) + \delta_t k(\frac{1}{10R_{11}C_4}X(n)Z(n) \\
&\quad + \frac{R_{15}}{R_{10}R_{14}C_4}W(n)), 1)
\end{aligned} \tag{7}$$

where k represents a time rescale factor ($k\tau = t; k = 100$). The circuit was simulated for the following values of parameters: $R_1 = R_2 = 2.5\text{K}\Omega$, $R_3 = R_6 = R_9 = 200\Omega$, $R_4 = 100\text{K}\Omega$, $R_5 = 1\text{K}\Omega$, $R_7 = 25\text{K}\Omega$, $R_8 = 33.2\text{K}\Omega$, $R_{10} = 66.9\text{K}\Omega$, $R_{11} = 1.6\text{K}\Omega$, $R_{12} = R_{13} = 5.6\text{K}\Omega$, $R_{14} = R_{15} = 560\Omega$, and $C_1 = C_2 = C_3 = C_4 = 100\text{nF}$. Depending on the chaos control parameter $c = \frac{1}{R_5C_2}$, the

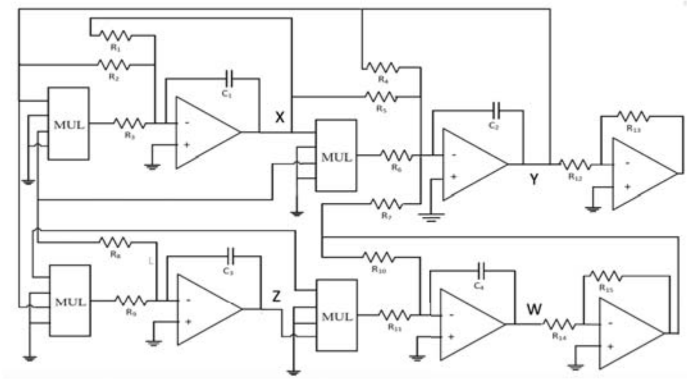

Fig. 4. Lorentz based chaotic circuit

circuit displays a rich variety of chaotic and hyperchaotic behavior. For instance, the circuit operates in hyperchaotic regime for $c = 90$ and in chaotic regime for $c = 275$. The time domain transient behavior of the Lorentz system circuit outputs X and Y for $c = 90$ (i.e., hyperchaotic regime) is depicted in Figure 5.

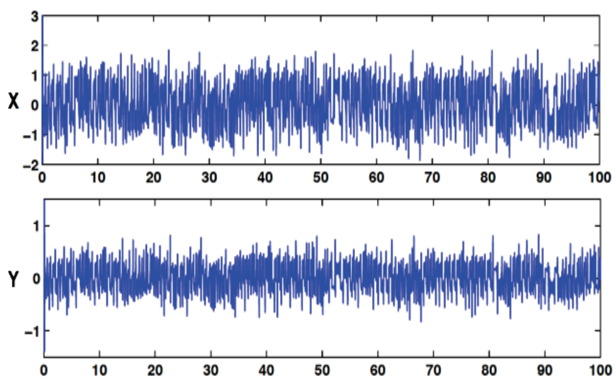

Fig. 5. Transient behavior of the Lorentz circuit X and Y outputs in hyperchaotic regime

It can be remarked that these outputs reveal a similar behavior to stochastic random noise. This demonstrates the need to assess the real source of random-like behavior observed in nonlinear circuits during the design process. Figure 6 shows the attractors of the Lorentz circuit during the chaotic regime (panel (a)) and the hyperchaotic regime (panel (b)). It can be noticed that the attractor structure of the hyperchaotic circuit behavior is more complex than the chaotic one. LZC measures acquired from the Lorentz circuit output X (dashed line) and 100 surrogates (dotted line) are shown in Figure 7 for chaotic behavior and in Figure 8 for hyperchaotic behavior. A good qualitative agreement between the results of our methodology and the analytical theory [16] is demonstrated; Indeed, the complexity of the original output (dashed line) is very different from its corresponding surrogates (dotted line) in the chaotic and hyperchaotic cases. This violates the hypothesis that the apparently random output is generated from a noisy circuit and hence indicates the deterministic structure of the circuit.

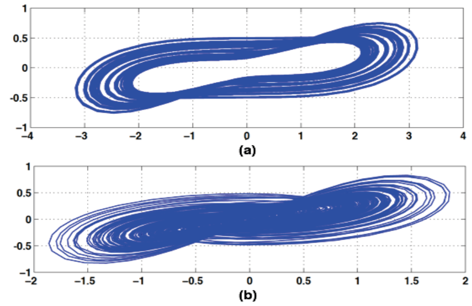

Fig. 6. Lorentz circuit attractors for chaotic (a) and hyperchaotic (b) regimes

Therefore, the proposed methodology was able to successfully distinguish the hyperchaos from stochastic behavior.

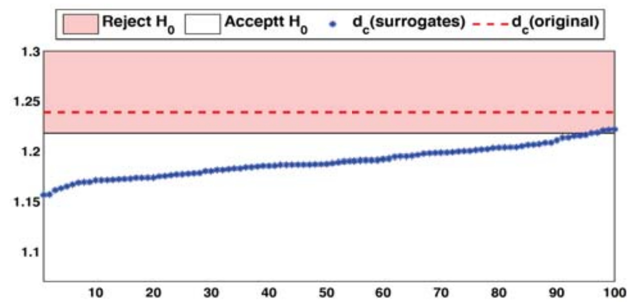

Fig. 7. Verification results for chaotic regime

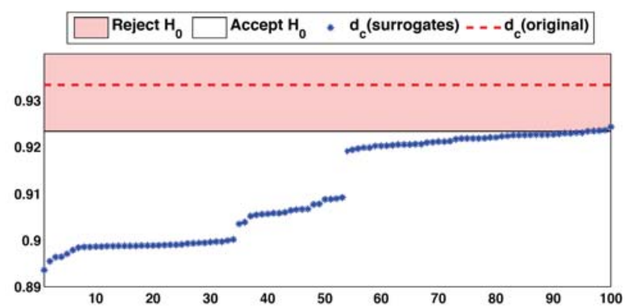

Fig. 8. Verification results for hyperchaotic regime

IV. Conclusion

This paper proposed a methodology based on the Lempel-Ziv Complexity measure to study aberrant analog circuit behavior due to chaos. The non-parametric Lempel-Ziv Complexity measure used as a test statistic offers a valuable means to predict actual circuit behavior (noisy vs chaotic). Conversely to other techniques, our approach is able to uncover noisy behavior even with non-Gaussian distribution and for hyperchaotic regimes. Experimental results on a first-order $\Sigma - \Delta$ modulator and a Lorentz based circuit demonstrate the efficiency of the proposed approach. The comparison of our results with the Gaussian Kernel test statistic shows that the latter is inappropriate for non-Gaussian noise type circuits. As a futur work, we plan to study other chaos measures like correlation dimensions on more complex analog circuits.

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