An Analytical Model for Performance Evaluation of Parallel Interference Cancellers in CDMA Systems

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Abstract: In multistage parallel interference cancellation, the tentative decisions for different users at each stage are mutually correlated, and they are also correlated with Gaussian noise. By taking the correlations into consideration and employing Price's theorem, an analytical model is proposed to refine the variance evaluation of the final decision variable. It is shown that the refined variance is very close to that obtained by simulation and can improve the accuracy in bit error rate evaluation.

1 Introduction

Performance analyses of various multistage parallel interference cancellation (PIC) detectors have been discussed in [1, 2]. As noticed [1, 2], there is a large discrepancy between the simulation and analytical results on the bit error rate (BER). One possible reason for such a difference is the inaccuracy in the variance evaluation of the decision variable due to the assumptions of (i) uncorrelated tentative decisions for different users and (ii) uncorrelation between the tentative decisions and Gaussian noise. By considering these correlations, in this letter, we present an analytical model to refine the variance evaluation of the final decision variable in the PIC. Price's theorem [3] is employed to compute the aforementioned correlations. Results show that a very accurate variance can be obtained and it improves the accuracy of the BER evaluation.

2 System Model

We consider a synchronous CDMA system with BPSK modulation. The received signal is given by

$$r(t) = \sum_{k=1}^{K} \alpha_k b_k(t) c_k(t) + \eta(t) \tag{1}$$

where $\alpha_k := \sqrt{E_b^{(k)}/T}$ is the amplitude of the k-th user. Here, $E_b^{(k)}$ is the bit energy and T is the bit interval. $b_k(t)$ and $c_k(t)$ denote the data and pseudo-random (PN) code waveforms of the k-th user, respectively. $\eta(t)$ is an additive white Gaussian noise (AWGN) with power spectral density $N_o/2$.

At the receiver, the despreading for the i-th user is performed as $y_i = \frac{1}{T}\int_0^T r(t)c_i(t)dt$, which can be expressed as

$$y_i = \alpha_i b_i + \sum_{k=1,k\neq i}^{K} \alpha_k \rho_{ik} b_k + z_i \tag{2}$$

where $\rho_{ik} = \frac{1}{T}\int_0^T c_i(t)c_k(t)dt$ is the cross-correlation between $c_i(t)$ and $c_k(t)$. $z_i = \frac{1}{T}\int_0^T c_i(t)\eta(t)dt$ is the resultant noise component.

Without loss of generality, we focus on the first user. A single-user receiver makes a decision as $\hat{b}_1 = \text{sgn}\{y_1\}$. It is well known that the single-user receiver suffers from the multiple access interference (MAI) (i.e., the sum term at the right hand-side of (2)). To suppress the MAI, PIC detectors [1, 2] have been proposed. In the PIC, the MAI will be reconstructed relying on the tentative decisions made in the previous stage and then subtracted from each user's received signal. Specifically, the final decision variable in a one-stage PIC is given by

$$y_1^{(1)} = y_1 - \sum_{k=2}^{K} \alpha_k \hat{b}_k^{(0)} \rho_{1k} \tag{3}$$

where $\hat{b}_k^{(0)} = \text{sgn}\{y_k\}$ denotes the tentative decision of user k in the previous (initial) stage. A final decision can be made based on $y_1^{(1)}$, namely, $\hat{b}_1^{(1)} = \text{sgn}\{y_1^{(1)}\}$.

3 Performance Analysis

To evaluate the BER performance, the decision variable $y_1^{(1)}$ can be approximated as a Gaussian distributed random variable [1, 2], and its variance is needed for evaluation. Conditioning on the data vector $\boldsymbol{b} = [b_1, b_2, \cdots, b_K]$, the variance of $y_1^{(1)}$ can be evaluated as follows

$$\begin{aligned}
\sigma^2_{y_1^{(1)}|\boldsymbol{b}} &= \text{var}\left\{ \sum_{k=2}^{K} \alpha_k \rho_{1k}[b_k - \hat{b}_k^{(0)}] + z_1 \middle| \boldsymbol{b} \right\} \\
&= \sum_{k=2}^{K} 4|\alpha_k|^2 \rho_{1k}^2 P_{ek}^{(0)} + \text{var}\{z_1|\boldsymbol{b}\} - 2\sum_{k=2}^{K} \alpha_k \rho_{1k} E\{z_1 \hat{b}_k^{(0)}|\boldsymbol{b}\} \\
&\quad - \sum_{n=2}^{K} \sum_{m=2,m\neq n}^{K} \alpha_m \alpha_n \rho_{1m} \rho_{1n} E\{b_m \hat{b}_n^{(0)} + b_n \hat{b}_m^{(0)} - \hat{b}_m^{(0)} \hat{b}_n^{(0)}|\boldsymbol{b}\}
\end{aligned} \tag{4}$$

where $P_{ek}^{(0)}$ is the BER of the k-th user in the initial stage.

To the best of our knowledge, in all existing analyses for the variance evaluation of $y_1^{(1)}$, only the first two terms at the right hand-side of (4) are considered, while the remaining terms, under the assumptions of uncorrelated tentative decisions $\{\hat{b}_n^{(0)}\}$ and uncorrelated z_1 and $\{\hat{b}_k^{(0)}\}$, are expected to be zero [1, 2]. However, as to be shown later, due to the possible correlations between the elements in $\{\hat{b}_n^{(0)}, n = 1, ..., K, z_1\}$ and the remaining terms are not trivial especially when the system load is high and the PN cross-correlation large. In the following, we will deal with those correlation terms by adopting Price's theorem [3].

3.1 Evaluation of $E\{z_1 \hat{b}_k^{(0)} | \boldsymbol{b}\}$

Define $I_1(\mu_1) = E\{z_1 \hat{b}_k^{(0)} | \boldsymbol{b}\}$, where $\mu_1 := E\{z_1(y_k - m_{y_k|\boldsymbol{b}})|\boldsymbol{b}\}$, of which $m_{y_k|\boldsymbol{b}}$ is the conditional mean of y_k. Applying Price's theorem [3], we have

$$\frac{\partial I_1(u_1)}{\partial \mu_1} = E\left\{\frac{\partial z_1 \partial[\text{sgn}(y_k)]|\boldsymbol{b}}{\partial z_1 \partial y_k}\right\} = E\left\{2\delta(y_k)|\boldsymbol{b}\right\} = 2f_{y_k|\boldsymbol{b}}(0) \tag{5}$$

where $f_{y_k|\boldsymbol{b}}(y) = N(m_{y_k|\boldsymbol{b}}; \sigma_{y_k|\boldsymbol{b}})$ is the conditional probability density function (pdf) of y_k. It can be readily shown that $\sigma^2_{y_k|\boldsymbol{b}} = \sigma_n^2$, where σ_n^2 is the variance of the noise component z_k.

From (5) it is straightforward that (note that $I(0) = 0$)

$$I_1(\mu_1) = 2f_{y_k|\boldsymbol{b}}(0)E\{z_1(y_k - m_{y_k|\boldsymbol{b}})|\boldsymbol{b}\} + I(0) = 2f_{y_k|\boldsymbol{b}}(0)\rho_{1k}\sigma_n^2 \tag{6}$$

3.2 Evaluation of $E\{b_m \hat{b}_n^{(0)} | \boldsymbol{b}\}$

Defining $I_2 = E\{b_m \hat{b}_n^{(0)} | \boldsymbol{b}\}$, we have

$$I_2 = b_m \int_{-\infty}^{\infty} \text{sgn}(y_n|\boldsymbol{b}) f_{y_n|\boldsymbol{b}}(y_n|\boldsymbol{b}) dy_n = b_m \left\{2Q\left(-\frac{m_{y_n|\boldsymbol{b}}}{\sigma_n}\right) - 1\right\} \tag{7}$$

where $Q(\cdot)$ is the complementary error function.

3.3 Evaluation of $E\{\hat{b}_m^{(0)} \hat{b}_n^{(0)} | \boldsymbol{b}\}$

Let $I_3(\mu_3) = E\{\hat{b}_m^{(0)} \hat{b}_n^{(0)} | \boldsymbol{b}\} = E\{\text{sgn}(y_m)\text{sgn}(y_n)|\boldsymbol{b}\}$, where μ_3 denotes the covariance of y_m and y_n. As y_m and y_n can be assumed jointly normal conditioning on $\boldsymbol{b}$, we can use Price's theorem and obtain

$$\frac{\partial I(\mu_3)}{\partial \mu_3} = E\left\{\left.\frac{\partial^2[\text{sgn}(y_m)\text{sgn}(y_n)]}{\partial y_m \partial y_n}\right| \boldsymbol{b}\right\} = E\{4\delta(y_m)\delta(y_n)|\boldsymbol{b}\} = 4f_{y_m,y_n|\boldsymbol{b}}(0,0) \tag{8}$$

where $f_{y_m,y_n|\boldsymbol{b}}(y_m, y_n)$ is the conditional joint normal pdf of y_m and y_n.

From (8) we obtain

$$I_3(\mu_3) = 4f_{y_m,y_n|\boldsymbol{b}}(0,0)\mu_3 + I_3(0) \tag{9}$$

where the covariance μ_3 can be computed as

$$\mu_3 = E\{y_m y_n|\boldsymbol{b}\} - E\{y_m|\boldsymbol{b}\}E\{y_n|\boldsymbol{b}\} = E\{z_m\}E\{z_n\} = \rho_{mn}\sigma_n^2 \tag{10}$$

In (9), $I_3(0)$ is evaluated as $I_3(0) = E\{\text{sgn}(y_m)|\boldsymbol{b}\}E\{\text{sgn}(y_n)|\boldsymbol{b}\}$ by noting that if $\mu_3 = 0$, y_m and y_n are independent. Thus, we have

$$I_3(0) = \left[2Q\left(-\frac{m_{y_m|\boldsymbol{b}}}{\sigma_{y_m|\boldsymbol{b}}}\right) - 1\right]\left[2Q\left(-\frac{m_{y_n|\boldsymbol{b}}}{\sigma_{y_n|\boldsymbol{b}}}\right) - 1\right] \tag{11}$$

3.4 BER Evaluation

Denoting the unconditional variance of $y_1^{(1)}$ as $\sigma^2_{y_1^{(1)}}$, it follows from (4) that

$$\sigma^2_{y_1^{(1)}} = \sum_i \sigma^2_{y_1^{(1)}|\boldsymbol{b}_i} p(\boldsymbol{b}_i) \tag{12}$$

where $p(\boldsymbol{b}_i)$ denotes the possibility for a specific vector in the space of $\boldsymbol{b}$.

When the interference is assumed to be Gaussian, the BER of the one-stage PIC for user 1 is given by

$$P_{e1}^{(1)} = Q\left(\frac{\alpha_1}{\sigma_{y_1^{(1)}}}\right) \tag{13}$$

4 Illustrative Results

Illustrative results on the variance of the decision variable $y_1^{(1)}$, obtained by simulation, "conventional" analysis [1, 2] (i.e., considering only the first two terms at the right hand-side of (4)), and "refined" analysis (i.e., presented in the previous section), for various cases are listed in Tables 1-3. We assume equal PN cross-correlation, i.e., $\rho_{ik} = \rho$ for all i and k, and equal bit energy, i.e., $E_b^{(k)} = E_b$ for all k. From the tables, it can be seen that the values obtained by the "refined" analysis (Ref) are very close to those measured by simulation (Sim) in all circumstances. In contrast, the "conventional" analysis (Conv) tends to underestimate the variance, especially when the number of users is large. For example, in Table 3, when E_b/N_0=10dB and $K = 20$, the variance from the "conventional" analysis is only 1/5 of the true value measured by simulation.

When the refined variance is used in (13), the accuracy on the resulting BER of a 1-stage PIC in AWGN channel is also improved as indicated in Figs 1-2. Fig. 1(a)-(c) show the performance comparison of the three approaches: simulation, "conventional", and "refined" analyses, for the case of 10 users and ρ=0.1, 0.15, and 0.20. When the MAI is small (i.e., ρ =0.1) the Gaussian noise becomes dominant and the three approaches provide almost the same results as shown in Fig. 1(a). When the MAI becomes larger, the "refined" analysis provides results closer to the simulation ones than the "conventional" counterpart as indicated in Fig. 1(b) and (c). However, it is noted that there is still a gap between the BER curves obtained by the "refined" analysis and simulation. This may be due to the inadequate assumption of Gaussian MAI model when the number of users is small (e.g., 10). To further investigate the effects of number of users on the validity of a Gaussian MAI model, we examine the case of 20 users. Results plotted in Fig. 2(a)-(c) show that the "refined" analysis offers the BER performance much closer to that measured by simulation than the "conventional" analysis as ρ increases (i.e., larger MAI). The gap between the BER curves obtained by the "refined" analysis and simulation in Fig. 2(a)-(c) is smaller than that in Fig. 1(a)-(c). This indicates that the assumption of Gaussian MAI model is more acceptable as the number of users increases.

5 Conclusions

In this paper, we suggested a refined analysis to evaluate the variance of the decision variable for the PIC detectors. Unlike previous performance analyses, the proposed method considers the correlations between the tentative decisions and their correlation with the Gaussian noise. By using Price's theorem, a very accurate variance of the final decision variable can be obtained and improves the accuracy of BER.

References

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Table 1: Variance of Decision Variable $y_1^{(1)}$ ($\rho = 0.1$)

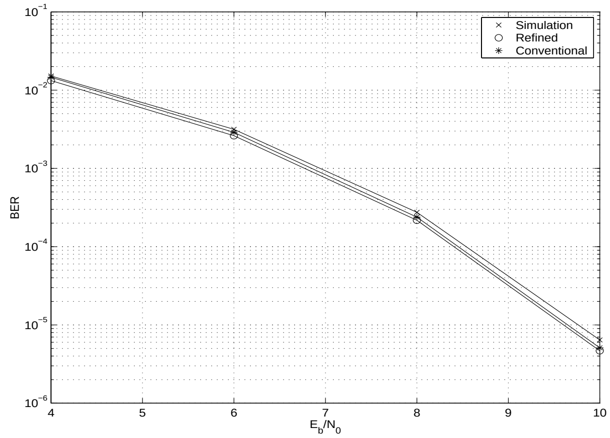
E_b/N_0	K	Sim	Anal (Ref)	Anal (Conv)
6	10	0.128127	0.128222	0.131222
6	20	0.180242	0.180514	0.154119
8	10	0.0806572	0.0807954	0.0819567
8	20	0.123137	0.123471	0.0997475
10	10	0.0507057	0.0509197	0.0513547
10	20	0.0873697	0.087767	0.0656662

Table 2: Variance of Decision Variable $y_1^{(1)}$ ($\rho = 0.15$)

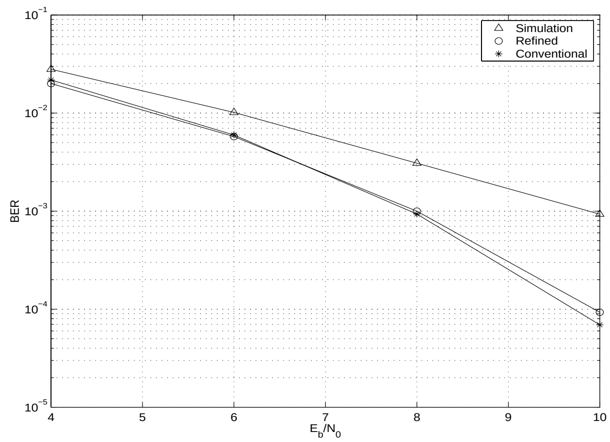
E_b/N_0	K	Sim	Anal (Ref)	Anal (Conv)
6	10	0.156363	0.156642	0.158335
6	20	0.673982	0.674381	0.278423
8	10	0.104273	0.104691	0.104691
8	20	0.618537	0.618652	0.216121
10	10	0.0709711	0.0715974	0.068866
10	20	0.590491	0.590216	0.176416

Table 3: Variance of Decision Variable $y_1^{(1)}$ ($\rho = 0.2$)

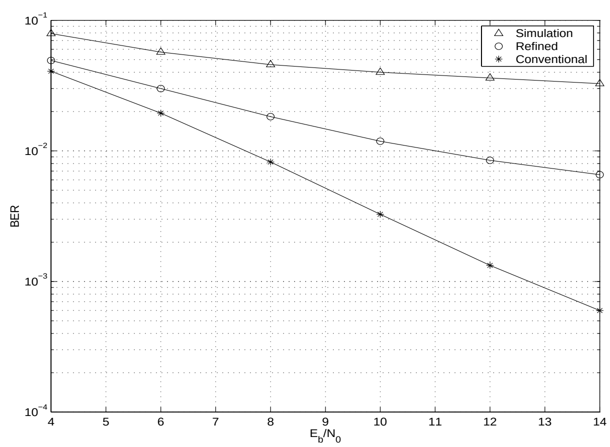
E_b/N_0	K	Sim	Anal (Ref)	Anal (Conv)
6	10	0.283549	0.282886	0.234513
6	20	2.21445	2.21559	0.563277
8	10	0.229595	0.228798	0.173807
8	20	2.214	2.21369	0.497271
10	10	0.196531	0.195465	0.135212
10	20	2.23417	2.23157	0.455111



(a) 10 users, $\rho = 0.1$

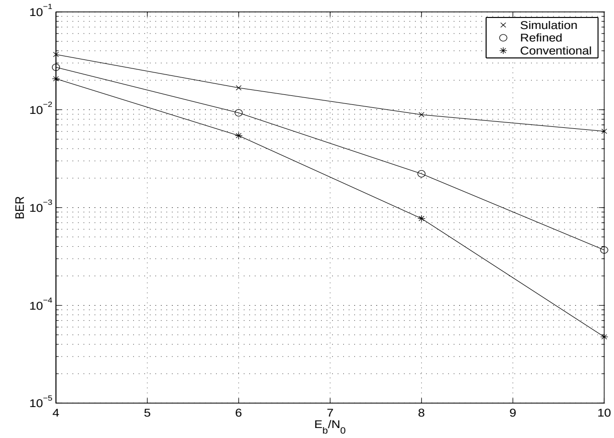


(b) 10 users, $\rho = 0.15$

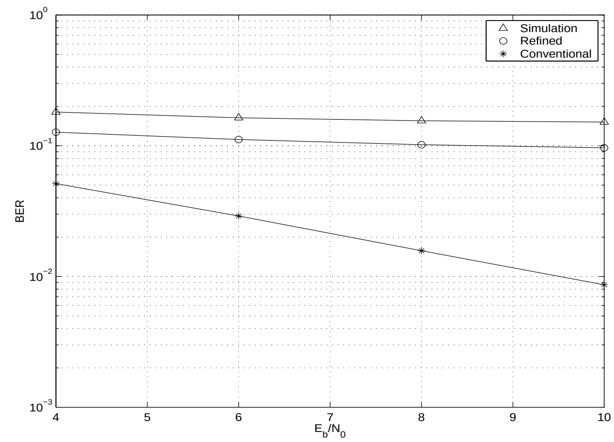


(c) 10 users, $\rho = 0.2$

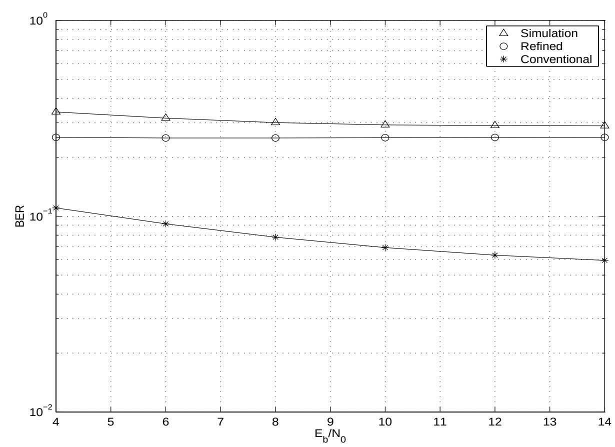
Fig. 1: BER of 1-stage PIC for 10 users.



(a) 20 users, $\rho = 0.1$



(b) 20 users, $\rho = 0.15$



(c) 20 users, $\rho = 0.2$

Fig. 2: BER of 1-stage PIC for 20 users.