Performance of Various Multistage Interference Cancellation Schemes for Asynchronous QPSK/DS/CDMA over Multipath Rayleigh Fading Channels *

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Abstract: Performance of multistage interference cancellation (MIC) and three combining techniques, i.e., multipath decorrelating (MIC-DECO), optimum combining (MIC-OPTM), and Rake combining (MIC-RAKE) for asynchronous QPSK/ DS/CDMA over frequency-selective multipath Rayleigh fading channels is studied. The analytical bit error probabilities of the MIC-DECO and MIC-OPTM are derived and shown to be in a good agreement with simulation results. Both analytical and simulation results show that the MIC-DECO, MIC-OPTM, and MIC-RAKE in a multi-user environment provide a good performance close to the ideal performance in a *single-user* system even in the presence of channel estimation error.

1 Introduction

Multistage interference cancellation (MIC) has been proposed to suppress the multiple-access interference (MAI) and the self interference (SI) for Direct-Sequence Code-Division Multiple Access (DS/ CDMA) systems in multipath fading channels [1–5], where to collect the fading replicas combining techniques can be used. To obtain a better understanding of various combining techniques, this paper presents a performance analysis of the MIC with three combining techniques, i.e., multipath decorrelating (MIC-DECO), optimum combining (MIC-OPTM) [6], and Rake combining (MIC-RAKE) [1] for a QPSK asynchronous DS/CDMA system over frequency-selective, Rayleigh multipath fading channels. The probabilities of bit error for the MIC-DECO and MIC-OPTM are derived by using the characteristic function method. In the derivation, Gaussian approximation method is used to model the multiuser and multipath interferences. The analysis also includes the effect of channel estimation error. Simulation results are shown in a good agreement with the analytical results. It is shown that the MIC-DECO can provide a good performance close to that of an ideal *single-user* system and outperforms the MIC with the *full* SI cancellation (MIC-FSI) [2, 3] even in the presence of the channel estima-

* This work is partially supported by Ericsson Research Canada

† J. F. Weng is with Sapphire R&D Inc., Canada, T. Le-Ngoc is with McGill University, Canada, G. Q. Xue is with Nortel Networks, Canada, and S. Tahar is with Concordia University, Canada

tion error. It is also found that in the initial stage, the MIC-DECO can provide a slightly better performance than the MIC-RAKE. However, in the *i*-th stage ($i > 0$), the MIC-RAKE has a performance close to that of the MIC-OPTM and superior to that of the MIC-DECO. Futhermore, the MIC-RAKE requires less computational complexity than the MIC-DECO. For this reason, the MIC-RAKE is a proper choice to be used to combat MAI and fading in CDMA systems.

2 System Description

Consider a QPSK asynchronous DS/CDMA system with K active users over a frequency-selective multipath Rayleigh fading channel. At the receiver, multistage interference cancellation is adopted to remove the multiple access interference (MAI) and the partial self-interference (PSI) [4]. Over L_u fading paths, the output signal in the i-th stage canceler can be written as,

$$\underline{V}_n^{(u)}(i) = \mathbf{R}_s \underline{\alpha}^{(u)} \sqrt{2\rho_u} e^{-j\phi_n^{(u)}} + \underline{\tilde{Si}}_n^{(u)}(i-1) + \underline{\tilde{M}}_n^{(u)}(i-1) + \underline{\eta}_n^{(u)} \tag{1}$$

where $\mathbf{R}_s$ is an $L_u \times L_u$ correlation matrix of the signature waveforms [4]. For each user, the corresponding correlation matrix $\mathbf{R}_s$ would be different. ρ_u is the normalized signal power, $\phi_n^{(u)}$ is the information bearing phase, and $\underline{\alpha}^{(u)} = [\alpha_1^{(u)}, \alpha_2^{(u)}, ..., \alpha_{L_u}^{(u)}]^T$ is the channel parameter vector. We assume the fading parameter $\alpha_l^{(u)}$ to be zero-mean, complex-valued Gaussian random variable with unit variance and mutually statistically independent for all u and l [2, 7]. The second and third terms on the right hand side of (1) denote the residual PSI and MAI, respectively [4]. $\underline{\eta}_n^{(u)} = [\eta_{n,1}^{(u)}, \eta_{n,2}^{(u)}, ..., \eta_{n,L_u}^{(u)}]^T$ is the noise vector with the correlation matrix $\mathbf{R}_s$ [4].

For the canceler output vector in (1), various combining techniques can be used to collect L_u fading replicas. A possible combining technique is to decorrelate this multipath signal first, i.e., $\mathbf{R}_s^{-1}\ \underline{V}_n^{(u)}(i)$, and then to combine the resulting signals. This approach is referred to as the MIC with decorrelating (MIC-DECO) [6]. The MIC-DECO is not optimum because the elements in the resultant noise vector, i.e.,

$$\underline{w}_n^{(u)}(i) = \mathbf{R}_s^{-1} \left[\underline{\tilde{Si}}_n^{(u)}(i-1) + \underline{\tilde{M}}_n^{(u)}(i-1) + \underline{\eta}_n^{(u)} \right] \tag{2}$$

are correlated. If we know the correlation *a priori*, the optimum combining can be used by whitening the noise before combining. Moreover, if the PSI and MAI are successfully removed, the noise vector in (2) reduces to $\underline{\eta}_n^{(u)}$. As a result, the optimum combining becomes a well-known Rake combining (MIC-RAKE) [1, 5, 7].

A general form for the combined output signal $X_n^{(u)}(i)$ in the i-th stage can be given by

$$X_n^{(u)}(i) = [\underline{\hat{\alpha}}^{(u)}]^H \mathbf{R} \underline{V}_n^{(u)}(i) \tag{3}$$

where $\underline{\hat{\alpha}}^{(u)}$ is the vector for the estimated channel parameter [2].

From the above discussion, the expression in (3) can reflect MIC-DECO, the MIC-OPTM, and the MIC-RAKE by setting the matrix $\mathbf{R}$ to $\mathbf{R}_s^{-1}$, $[\mathbf{R}_w^{(u)}(i)\mathbf{R}_s]^{-1}$, and $\mathbf{I}$ (the identity matrix), respectively. Here, $\mathbf{R}_w^{(u)}(i)$

denotes the correlation matrix of the noise vector $\underline{w}_n^{(u)}(i)$ in (2). Among the three schemes, the MIC-RAKE has the simplest structure. Finally, the decision is made as follows. After computing the following decision variables for QPSK (M=4), $D_p(i) = \text{Re}\left\{X_n^{(u)}(i)\, e^{j\psi_p}\right\}$, where $\psi_p := 2\pi(p-1)/M, p = 1, 2, ..., M$, the symbol is decided in favor of $\hat{\phi}_n^{(u)}(i) = \psi_p = \text{argmax}\,\{D_p(i)\}$.

3 Performance Analysis

Without loss of generality, we assume that all the symbols are transmitted with equal probability and consider user u as the user of interest with $\phi_n^{(u)}$=0. For the sake of notational simplicity, the indices *n, u, i* of $X_n^{(u)}(i)$ are omitted whenever there is no ambiguity.

We first consider a general combining output, i.e.,

$$X = [C + G]^H [C\sqrt{2\rho} + Z] \tag{4}$$

where ρ denotes the normalized signal power, C denotes the L-dimension signal vector, G and Z denote two L-dimension noise vectors. The elements in each vector of C, G, and Z are statistically independent and the vectors C, G, and Z are mutually statistically independent. We let $\frac{1}{2}E\{CC^H\}$=$\Lambda^{(C)} = diag(\lambda_1^{(C)}, \lambda_2^{(C)}, ..., \lambda_L^{(C)})$, $\frac{1}{2}E\{GG^H\}$=$\Lambda^{(G)} = diag(\lambda_1^{(G)}, \lambda_2^{(G)}, ..., \lambda_L^{(G)})$, $\frac{1}{2}E\{ZZ^H\}$=$\Lambda^{(Z)} = diag(\lambda_1^{(Z)}, \lambda_2^{(Z)}, ..., \lambda_L^{(Z)})$,.

As a result, with Gray coding, the bit error rate (BER) of QPSK can be given by [8]

$$P_b(\Lambda^{(C)}, \Lambda^{(G)}, \Lambda^{(Z)}) = -\sum_{\{\omega_p\}} \text{Res}\left[\Phi_{\gamma=-\pi/4}(\omega)/\omega, \omega_p\right] \tag{5}$$

where $\Phi_\gamma(\omega)$ denotes the CF of Re$\left\{X\, e^{j\gamma}\right\}$. $\{\omega_p\}$ denotes the set of poles of $\Phi_\gamma(\omega)/\omega$ in the upper half plane (Im(ω)>0) and Res$\,[f(\omega), \omega_p]$ the residue of $f(\omega)$ at $\omega = \omega_p$. By using Eq.B-5 of [7], the CF of Re$\{X e^{j\gamma}\}$ can be derived as

$$\Phi_\gamma(\omega) = \prod_{q=1}^{L} \frac{1}{a_q^2\omega^2 - 2j\omega b_q + 1} \tag{6}$$

where $a_q := \sqrt{[\lambda_q^{(Z)} + 2\rho\lambda_q^{(G)}]\lambda_q^{(C)} + \lambda_q^{(Z)}\lambda_q^{(G)}}$ and $b_q := \sqrt{2\rho}\, cos\gamma\, \lambda_q^{(C)}$.

Next, we will use the formula in (5) to evaluate the performance of the MIC-DECO and MIC-OPTM.

In the MIC-DECO scheme, the combining output can still be obtained by (4) with $C = \underline{\alpha}^{(u)}$, $G = \Delta\underline{\alpha}^{(u)} = \hat{\underline{\alpha}}^{(u)} - \underline{\alpha}^{(u)}$, which denotes the vector of the estimate error, and Z=$\underline{w}_n^{(u)}(i)$. As mentioned previously, the noise vector $\underline{w}_n^{(u)}(i)$ has a correlation matrix denoted by $\mathbf{R}_w^{(u)}(i)$. The evaluation of $\mathbf{R}_w^{(u)}(i)$ is relying on the error probability evaluated in the previous stage and the Gaussian approximation of PSI and MAI as shown in [3, 4]. By diagonalizing $\mathbf{R}_w^{(u)}(i)$=$\mathbf{Q}_w(i)\Lambda_w(i)\mathbf{Q}_w^H(i)$ for orthogonal $\mathbf{Q}_w(i)$ and diagonal $\Lambda_w(i)$, the combiner output of the MIC-DECO will have the same form of (4) but with C=$\mathbf{Q}_w^H(i)\underline{\alpha}^{(u)}$, G=$\mathbf{Q}_w^H(i)\Delta\underline{\alpha}^{(u)}$, and Z=$\mathbf{Q}_w^H(i)\underline{w}_n^{(u)}(i)$. In order to yield a mathematically tractable analysis, we assume independent and identically distributed (i.i.d.) $\alpha_l^{(u)}$ and i.i.d. $\Delta\alpha_l^{(u)}$ for all l [2, 3, 7]. As a result, we have $\frac{1}{2}E\{CC^H\}$=$\sigma_\alpha^2\mathbf{I}$, $\frac{1}{2}E\{GG^H\}$= $\sigma_e^2\mathbf{I}$, and $\frac{1}{2}E\{ZZ^H\}$=$\Lambda_w(i)$, where σ_α^2 and σ_e^2 are the variances of $\{\alpha_l^{(u)}\}$ and $\{\Delta\alpha_l^{(u)}\}$, respectively. By applying the formula of (5), the BER of the MIC-DECO is given by

$$P_{b,DECO}(i) = P_b(\sigma_\alpha^2\mathbf{I}, \sigma_e^2\mathbf{I}, \Lambda_w(i)) \tag{7}$$

In the MIC-OPTM scheme, the combiner output is given by (3) with $\mathbf{R} = [\mathbf{R}_w^{(u)}(i)\mathbf{R}_s]^{-1}$. Again, by using the diagonalization of $\mathbf{R}_w^{(u)}(i)$, the combiner output can be written in (4) with C=$\Lambda_w^{-\frac{1}{2}}(i)\mathbf{Q}_w^H(i)\underline{\alpha}^{(u)}$, G=$\Lambda_w^{-\frac{1}{2}}(i)\mathbf{Q}_w^H(i)\Delta\underline{\alpha}^{(u)}$, and Z=$\Lambda_w^{-\frac{1}{2}}(i)\mathbf{Q}_w^H(i)\underline{w}_n^{(u)}(i)$. Here, we still consider i.i.d. $\alpha_l^{(u)}$ and i.i.d. $\Delta\alpha_l^{(u)}$ for all l. We have $\frac{1}{2}E\{CC^H\}$= $\sigma_\alpha^2\Lambda_w^{-1}(i)$, $\frac{1}{2}E\{GG^H\}$= $\sigma_e^2\Lambda_w^{-1}(i)$, and $\frac{1}{2}E\{ZZ^H\}$=$\mathbf{I}$. By using (5), the BER of the MIC-OPTM is given by

$$P_{b,OPTM}(i) = P_b(\sigma_\alpha^2\Lambda_w^{-1}(i), \sigma_e^2\Lambda_w^{-1}(i), \mathbf{I}) \tag{8}$$

It is noteworthy to mention that $\Lambda_w(i)$ in (8) is different from that in (7) except for i=0 (in the initial stage).

4 Illustrative Results

This section presents several numerical and simulation results on the BER performance of the MIC-DECO, MIC-OPTM, and MIC-RAKE in QPSK asynchronous CDMA system over multipath Rayleigh fading channels. For comparison, the BERs of the MIC with the full SI cancellation (MIC-FSI) [2, 3] are also provided. Without loss of generality, we assume the first user to be the user of interest.

We consider Gold sequences with N=31 as the PN codes and identical fading channels (1/2E$\{[\alpha_l^{(u)}]^2\} = 1$ for all fading paths). We set SNR#u per bit= $\frac{1}{2}L_u 2\rho_u$E$\{[\alpha_l^{(u)}]^2\}/\ log_2 M$= $\rho_u L_u$, and $L_u = L$. The time delay for user k in the l-th path is $\tau_l^{(u)}$=$T^{(u)}$+$lT_c/2$.

In the following, we first consider the performance of the MIC with perfect channel knowledge, where we let MIC-i denote the i-th stage MIC.

4.1 Perfect channel knowledge

Fig. 1 plots the analytical (anal) and simulation (sim) results on the BER of the MIC-DECO with the stage i=0,1 in a 3-user system with SNR=20dB over a 2-path frequency-selective identical Rayleigh fading channel. Equal-power users are considered. The analytical results for the MIC-DECO are evaluated by (7). For comparison, the simulation results of the MIC-FSI-1 and the MIC-RAKE-0,1, and the analytical results of the MIC-OPTM-0 evaluated via (8) are plotted. It can be seen that both the MIC-DECO-1 and MIC-FSI-1 can provide a significant performance improvement over the MIC-DECO-0 and the MIC-RAKE-0, in which no interference cancellation scheme is used. In addition, the MIC-DECO-1 can provide an improvement in SNR of about 2.5dB at a BER of

10^{-4}, as compared to the MIC-FSI-1. Similar results were obtained by the MIC-RAKE [4]. From the figure, we also note that the analytical results agree well with simulation ones especially for the case of the MIC-DECO-0. Apart from this, we can see that in the multiuser system ($K = 3$) and for $i > 0$, due to the effective subtraction of the interference, the canceler output is similar to that in the single-user system and thus the MIC-RAKE-1 conducts better than the MIC-DECO-1 as shown in the figure. However, the performance difference is very small (approximately 0.5dB at the level of BER=10^{-4}). When there is no interference cancellation (at the stage 0), both the MIC-RAKE-0 and the MIC-DECO-0 are not optimum and the MIC-DECO-0 has a slightly better performance than the MIC-RAKE-0 at high SNR (SNR>15dB).

In Fig. 2, we plot the analytical and simulation results on the BER of the MIC-DECO and the MIC-OPTM in a multi-user system with SNR=20dB over a 2-path Rayleigh fading channel versus the number of users. It shows that in the presence of multiple users, the MIC-DECO-1,2 still outperforms the detector without interference cancellation (MIC-DECO-0) and provides a performance close to that of the MIC-OPTM-1,2. Besides, it can be seen that for stage $i > 0$, the analytical results agree well with the simulation ones for small K (number of users). As K increases, deviation may occur. A possible explanation for this is that when K becomes large, the MAI also increases and thus the correlation between the decision variables in the previous stage becomes larger. When we ignore such correlation in evaluating the covariance matrices of the residual interference [2–4], performance difference occurs especially for large K. Note that in the 0-th stage, no such problem occurs because there is no interference cancellation.

4.2 Effect of channel estimation error

We assume the variance of the channel estimation to be the same for all l and k, and denoted by σ_e^2.

In Fig. 3, we consider the MIC-DECO and the MIC-RAKE in a 3-user system with SNR=20dB over L-path Rayleigh fading channels. σ_e^2=0.01. We assume that the detector knows the number of paths so that L-diversity combining is applied. As shown in this figure, analytical results of the MIC-DECO are in a very good agreement with those obtained by simulation. Comparing the performance of the MIC-DECO and MIC-RAKE, we can see that in the 0-th stage, both schemes can provide a similar performance, while in multi-stage cases ($i > 0$), the MIC-RAKE-i outperforms the MIC-DECO-i, especially for large L.

Finally, we consider long PN codes for each user where the PN sequence is the combination of a long PN sequence plus two in-phase and quadrature PN sequences as specified in [9]. Unlike the Gold sequence, the long PN sequence is time-varying and different from symbol to symbol. The number of chips per symbol is still set to $N = 31$. Here, we focus on the MIC-RAKE since it has a better performance and lower complexity than the MIC-DECO. The results for the MIC-FSI and MIC-RAKE in a multiuser system over a 2-path Rayleigh fading channel are shown in Fig. 4. Perfect channel knowledge is assumed. From Fig. 4, we can see that with the long PN codes, the MIC-RAKE outperforms the MIC-FSI.

5 Conclusions

This paper studies the performance of the MIC-DECO, MIC-OPTM, and MIC-RAKE. It is shown that the MIC-DECO provides a good performance, close to the ideal single-user performance even in the presence of channel estimation error. If the correlation of the noise elements among the fading replicas is known *a priori*, the MIC-OPTM can be achieved. If the multiuser and multipath interferers are effectively subtracted, the MIC-OPTM can be well approximated by the MIC-RAKE with reduced complexity.

References

[1] S. Striglis, A. Kaul, N. Yang, and B. D. Woerner, "A multistage RAKE receiver for improved capacity of CDMA systems," in *Proc. IEEE Vehi. Tech. Conf., VTC94, Stockholm, Sweden*, pp. 789–793, June 1994.

[2] Y. C. Yoon, R. Kohno, and H. Imai, "A spread-spectrum multiaccess system with cochannel interference cancellation for multipath fading channels," *IEEE J. Select. Areas Commun.*, vol. SAC-11, pp. 1067–1075, Sept. 1993.

[3] A. L. C. Hui and K. B. Letaief, "Successive interference cancellation for multiuser asynchronous DS/CDMA detectors in multipath fading links," *IEEE Trans. Commun.*, vol. COM-46, pp. 384–391, Mar. 1998.

[4] J. F. Weng, G. Q. Xue, T. Le-Ngoc, and S. Tahar, "Multistage interference cancellation with diversity reception for asynchronous QPSK DS/CDMA systems over multipath fading channels," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 2162–2180, Dec. 1999.

[5] D. Divsalar, M. Simon, and D. Raphaeli, "Improved parallel interference cancellation for CDMA," *IEEE Trans. Commun.*, vol. COM-46, pp. 258–268, Feb. 1998.

[6] J. F. Weng, G. Q. Xue, T. Le-Ngoc, and S. Tahar, "Multistage interference cancellation with multipath decorrelating for QPSK asynchronous DS/CDMA system over multipath fading," in *Proc. IEEE Vehi. Tech. Conf., VTC99, Houston, Texas*, May 1999.

[7] J. G. Proakis, *Digital communications*. New York: McGraw-Hill, 3rd Ed, 1995.

[8] P. Y. Kam, "Bit error probability of MDPSK over the non-selective Rayleigh fading channels with diversity reception," *IEEE Trans. Commun.*, vol. COM-39, pp. 220–224, Feb. 1991.

[9] TIA/EIA/IS-95, "Mobile station base station compatability standard for dual-mode wideband spread spectrum cellular system," in *Telecommun. Industry Association Standard*, TIA/EIA, Jul. 1993.

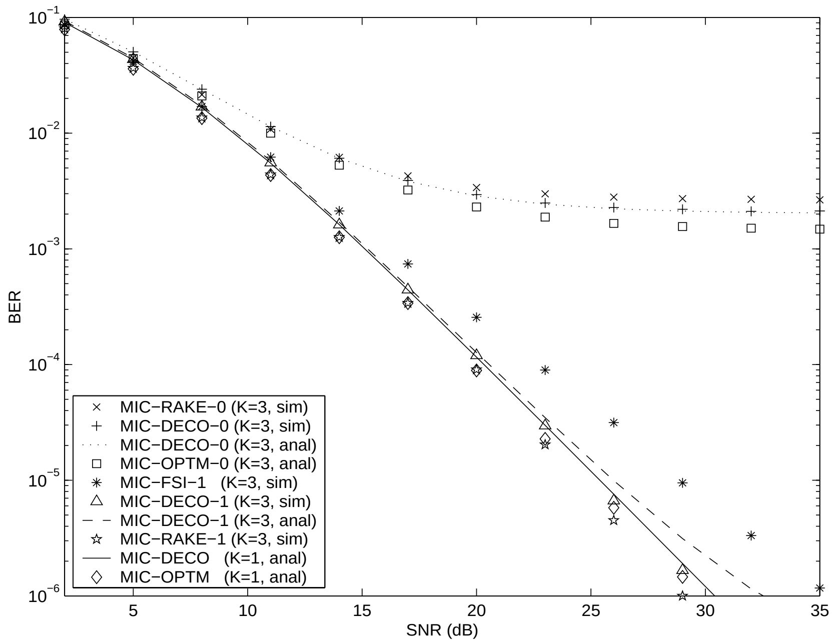

Fig. 1: BERs of the MIC schemes in a 3-user system over a 2-path Rayleigh fading channel. (N=31, equal-power users. Perfect channel knowledge).

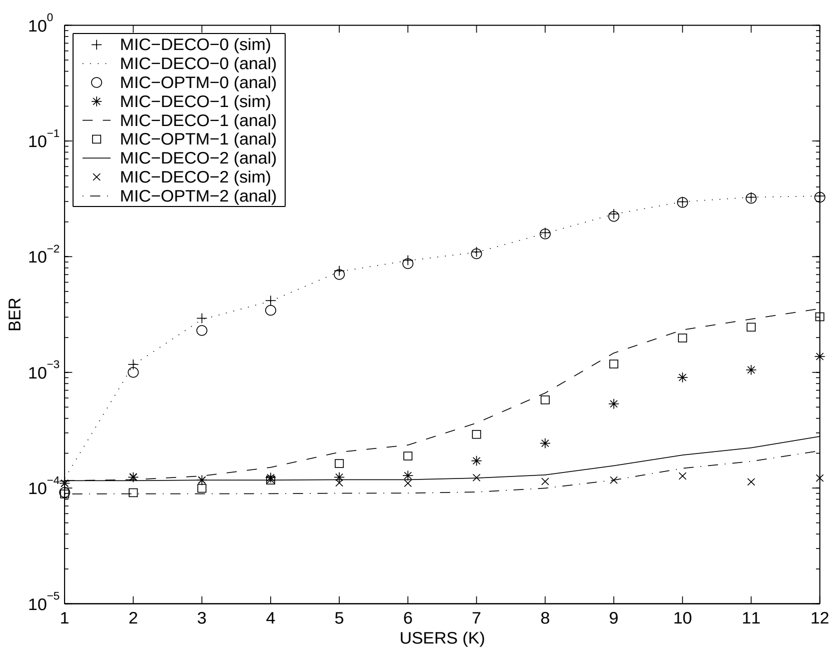

Fig. 2: BERs of the MIC-DECO and MIC-OPTM in a multi-user system over a 2-path Rayleigh fading channel. (SNR=20dB, N=31, equal-power users. Perfect channel knowledge).

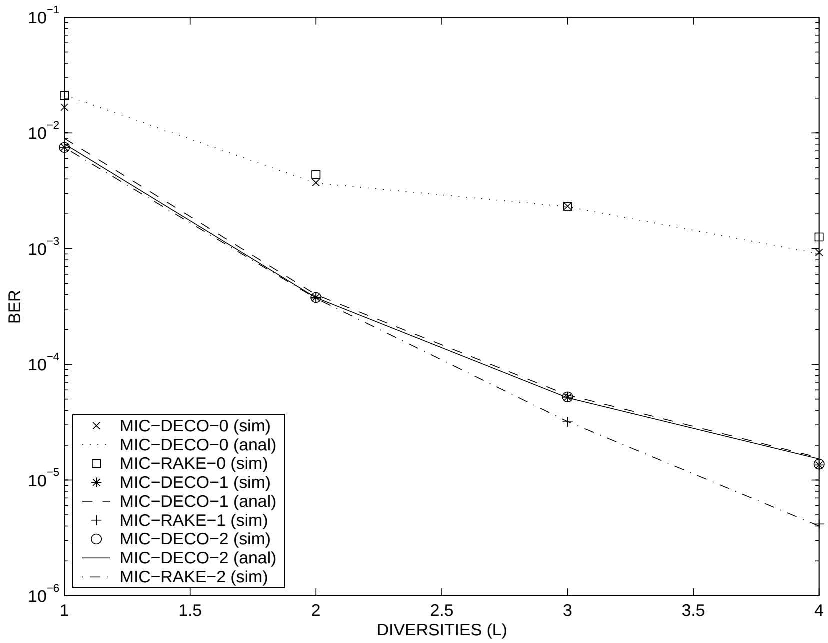

Fig. 3: BERs of the MIC-DECO and MIC-RAKE in a 3-user system over L-path (L=1,2,3,4) Rayleigh fading channels. (SNR=20dB, N=31, equal-power users. Imperfect channel knowledge, $\sigma_e^2 = 10^{-2}$.)

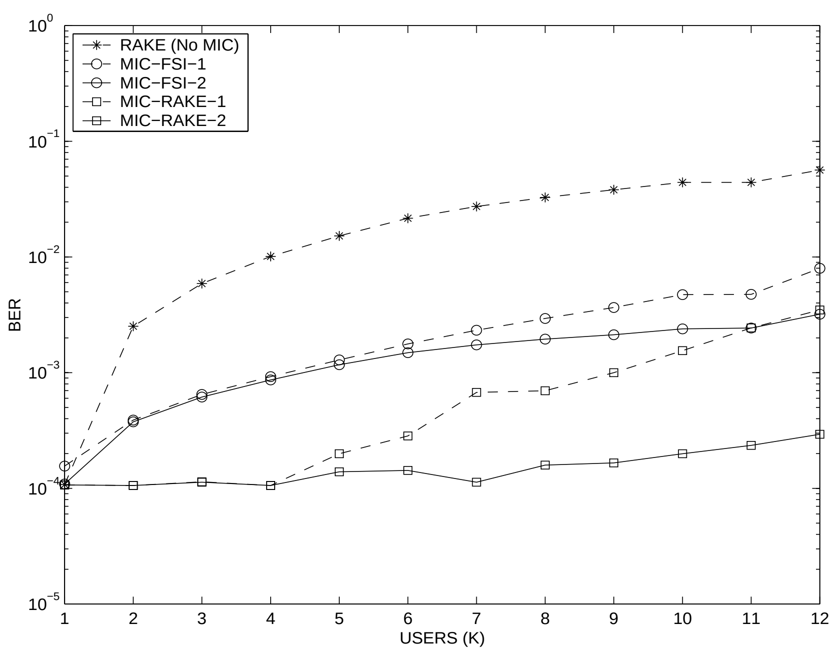

Fig. 4: Simulation results on the BER of the MIC-FSI and MIC-RAKE in a multiuser system over a 2-path Rayleigh fading channel. (SNR=20dB, N=31, perfect channel knowledge, long PN sequence.)