1 CAS_general Theory

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Parent Theories: prob_PAND_CSP

1.1 Definitions

[UNIONL_def]

$\vdash$ (UNIONL [] = $\{\}$) $\wedge$ $\forall s\ ss.$ UNIONL (s::ss) = $s \cup$ UNIONL ss

1.2 Theorems

[ALL_DISTINCT_RVg_def]

```
⊢ ALL_DISTINCT_RVg
    [A0; A1; A2; A3; A4; A5; A6; A7; A8; A9; A10] p t ⟺
  All_distinct_events p
    [A6; A7; D_AND A4 (D_BEFORE A3 A4);
     D_AND A5 (D_BEFORE A4 A5);
     D_OR (D_AND A9 (D_BEFORE A8 A9))
       (D_AND A8 (D_BEFORE A10 A8)); D_AND (D_AND A0 A1) A2] t ∧
  rv_gt0_ninfinity
    [A0; A1; A2; A3; A4; A5; A6; A7; A8; A9; A10] ∧
  ∀s.
     ALL_DISTINCT
       (MAP (λa. a s)
          [A0; A1; A2; A3; A4; A5; A6; A7; A8; A9; A10])
```

[ALL_DISTINCT_RVg_ind]

```
⊢ ∀P.
      (∀A0 A1 A2 A3 A4 A5 A6 A7 A8 A9 A10 p t.
           P [A0; A1; A2; A3; A4; A5; A6; A7; A8; A9; A10] p t) ∧
      (∀v4. P [] (FST v4) (SND v4)) ∧
      (∀v7 v8. P [v7] (FST v8) (SND v8)) ∧
      (∀v12 v11 v13. P [v12; v11] (FST v13) (SND v13)) ∧
      (∀v18 v17 v16 v19. P [v18; v17; v16] (FST v19) (SND v19)) ∧
      (∀v25 v24 v23 v22 v26.
           P [v25; v24; v23; v22] (FST v26) (SND v26)) ∧
      (∀v33 v32 v31 v30 v29 v34.
           P [v33; v32; v31; v30; v29] (FST v34) (SND v34)) ∧
      (∀v42 v41 v40 v39 v38 v37 v43.
           P [v42; v41; v40; v39; v38; v37] (FST v43) (SND v43)) ∧
      (∀v52 v51 v50 v49 v48 v47 v46 v53.
           P [v52; v51; v50; v49; v48; v47; v46] (FST v53)
             (SND v53)) ∧
      (∀v63 v62 v61 v60 v59 v58 v57 v56 v64.
           P [v63; v62; v61; v60; v59; v58; v57; v56] (FST v64)
             (SND v64)) ∧
```

```
(∀ v75 v74 v73 v72 v71 v70 v69 v68 v67 v76.
     P [v75; v74; v73; v72; v71; v70; v69; v68; v67]
       (FST v76) (SND v76)) ∧
(∀ v88 v87 v86 v85 v84 v83 v82 v81 v80 v79 v89.
     P [v88; v87; v86; v85; v84; v83; v82; v81; v80; v79]
       (FST v89) (SND v89)) ∧
(∀ v106 v105 v104 v103 v102 v101 v100 v99 v98 v97 v96 v92
     v93 v107.
     P
       (v106::v105::v104::v103::v102::v101::v100::v99::
          v98::v97::v96::v92::v93) (FST v107)
       (SND v107)) ⇒
∀ v v1 v2. P v v1 v2
```

[CAS_general_final]

```
⊢ ∀ PA PB PS MS MA MB B_a B_d CS SS P B.
    (∀ s.
         ALL_DISTINCT
           [MA s; MS s; PA s; PB s; PS s; MB s; P s; B_d s;
            B_a s; CS s; SS s]) ∧ (D_BEFORE B_a P = NEVER) ⇒
    (D_OR
       (D_OR
          (D_AND (shared_spare PA PB PS PS)
             (shared_spare PB PA PS PS))
          (D_OR (P_AND MS MA) (CSP MA MB)))
       (WSP (FDEP (D_OR CS SS) P) (FDEP (D_OR CS SS) B_a)
          (FDEP (D_OR CS SS) B_d)) =
     D_OR CS
       (D_OR SS
          (D_OR (D_AND MA (D_BEFORE MS MA))
             (D_OR (D_AND MB (D_BEFORE MA MB))
                (D_OR (D_AND B_a (D_BEFORE P B_a))
                   (D_OR (D_AND P (D_BEFORE B_d P))
                      (D_AND (D_AND PA PB) PS)))))))
```

[CAS_general_PROB]

```
⊢ ∀ CS SS MA MS MB P B_a B_d PA PB PS p t f_MA f_BP f_P
    f_cond_P_B f_MB_MA f_cond_MA_MB.
    0 ≤ t ∧ prob_space p ∧ (D_BEFORE B_a P = NEVER) ∧
    ALL_DISTINCT_RVg
      [PA; PB; PS; MS; MA; MB; CS; SS; P; B_a; B_d] p t ∧
    indep_vars_setsg
      [PA; PB; PS; MS; MA; MB; CS; SS; P; B_a; B_d] p t ∧
    DISJOINT_WSP P B_a B_d t ∧
    (∀ x y.
         cond_density lborel lborel p (λ s. real (B_a s))
           (λ s. real (P s)) y f_BP f_P f_cond_P_B) ∧
    den_gt0_ninfinity f_BP f_P f_cond_P_B ∧
    cont_CDF p (λ s. real (B_d s)) ∧
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```
measurable_CDF p (λs. real (B_d s)) ∧
(∀x y.
     cond_density lborel lborel p (λs. real (MB s))
       (λs. real (MA s)) y f_MB_MA f_MA f_cond_MA_MB) ∧
den_gt0_ninfinity f_MB_MA f_MA f_cond_MA_MB ∧
indep_varp p (pair_measure lborel lborel)
  (λx. (real (MB x),real (MA x))) lborel
  (λx. real (MS x)) ∧ (∀x. f_MB_MA x ≠ PosInf) ∧
cont_CDF p (λs. real (MS s)) ∧
measurable_CDF p (λs. real (MS s)) ⇒
(prob p
   (DFT_event p
      (D_OR
         (D_OR
            (D_AND (shared_spare PA PB PS PS)
               (shared_spare PB PA PS PS))
            (D_OR (P_AND MS MA) (CSP MA MB)))
         (WSP (FDEP (D_OR CS SS) P)
            (FDEP (D_OR CS SS) B_a)
            (FDEP (D_OR CS SS) B_d))) t) =
 CDF p (λs. real (CS s)) t + CDF p (λs. real (SS s)) t +
 pos_fn_integral lborel
   (λy.
        f_MA y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (MS s)) y)) +
 pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_MA_MB y x)) +
 (pos_fn_integral lborel
    (λy.
         indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
         pos_fn_integral lborel
           (λx.
                indicator_fn {w | y < w ∧ w ≤ t} x ×
                f_cond_P_B y x)) +
  pos_fn_integral lborel
    (λy.
         f_P y ×
         (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
          CDF p (λs. real (B_d s)) y))) +
 CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
 CDF p (λs. real (PS s)) t -
 CDF p (λs. real (CS s)) t × CDF p (λs. real (SS s)) t -
 CDF p (λs. real (CS s)) t ×
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```
pos_fn_integral lborel
  (λ y.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λ s. real (MS s)) y)) -
CDF p (λ s. real (CS s)) t ×
pos_fn_integral lborel
  (λ y.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λ x.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) -
CDF p (λ s. real (CS s)) t ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) -
CDF p (λ s. real (CS s)) t × CDF p (λ s. real (PA s)) t ×
CDF p (λ s. real (PB s)) t × CDF p (λ s. real (PS s)) t -
CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λ s. real (MS s)) y)) -
CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λ x.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) -
CDF p (λ s. real (SS s)) t ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
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              f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) -
CDF p (λs. real (SS s)) t × CDF p (λs. real (PA s)) t ×
CDF p (λs. real (PB s)) t × CDF p (λs. real (PS s)) t -
pos_fn_integral lborel
  (λy.
       f_MA y × CDF p (λs. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λx.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) -
pos_fn_integral lborel
  (λy.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λs. real (MS s)) y)) ×
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) -
pos_fn_integral lborel
  (λy.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λs. real (MS s)) y)) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t -
pos_fn_integral lborel
  (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λx.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) ×
(pos_fn_integral lborel
   (λy.
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        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) -
pos_fn_integral lborel
  (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λx.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t -
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
CDF p (λs. real (CS s)) t × CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λs. real (MS s)) y)) +
CDF p (λs. real (CS s)) t × CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λx.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) +
CDF p (λs. real (CS s)) t × CDF p (λs. real (SS s)) t ×
(pos_fn_integral lborel
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    (λ y.
         indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
         pos_fn_integral lborel
           (λ x.
                indicator_fn {w | y < w ∧ w ≤ t} x ×
                f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) +
CDF p (λ s. real (CS s)) t × CDF p (λ s. real (SS s)) t ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t +
CDF p (λ s. real (CS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y × CDF p (λ s. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λ x.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) +
CDF p (λ s. real (CS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λ s. real (MS s)) y)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) +
CDF p (λ s. real (CS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λ s. real (MS s)) y)) ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t +
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CDF p (λs. real (CS s)) t ×
pos_fn_integral lborel
  (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λx.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) ×
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) +
CDF p (λs. real (CS s)) t ×
pos_fn_integral lborel
  (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λx.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
CDF p (λs. real (CS s)) t ×
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
       f_MA y × CDF p (λs. real (MS s)) y ×
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      indicator_fn {y′ | 0 ≤ y′ ∧ y′ ≤ t} y ×
      pos_fn_integral lborel
        (λ x.
             f_cond_MA_MB y x ×
             indicator_fn {x′ | y < x′ ∧ x′ ≤ t} x)) +
CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λ s. real (MS s)) y)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) +
CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λ s. real (MS s)) y)) ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t +
CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λ x.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
```

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        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) +
CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λx.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
CDF p (λs. real (SS s)) t ×
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
pos_fn_integral lborel
  (λy.
       f_MA y × CDF p (λs. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λx.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) +
pos_fn_integral lborel
  (λy.
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        f_MA y × CDF p (λs. real (MS s)) y ×
        indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
        pos_fn_integral lborel
          (λx.
               f_cond_MA_MB y x ×
               indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
pos_fn_integral lborel
  (λy.
        f_MA y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (MS s)) y)) ×
(pos_fn_integral lborel
   (λy.
         indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
         pos_fn_integral lborel
           (λx.
                indicator_fn {w | y < w ∧ w ≤ t} x ×
                f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
         f_P y ×
         (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
          CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
pos_fn_integral lborel
  (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_MA_MB y x)) ×
(pos_fn_integral lborel
   (λy.
         indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
         pos_fn_integral lborel
           (λx.
                indicator_fn {w | y < w ∧ w ≤ t} x ×
                f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
         f_P y ×
         (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
          CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t -
CDF p (λs. real (CS s)) t × CDF p (λs. real (SS s)) t ×
```

```
pos_fn_integral lborel
  (λ y.
       f_MA y × CDF p (λ s. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λ x.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) -
CDF p (λ s. real (CS s)) t × CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λ s. real (MS s)) y)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) -
CDF p (λ s. real (CS s)) t × CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λ s. real (MS s)) y)) ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t -
CDF p (λ s. real (CS s)) t × CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λ x.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
```

```
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) -
CDF p (λ s. real (CS s)) t × CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λ x.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t -
CDF p (λ s. real (CS s)) t × CDF p (λ s. real (SS s)) t ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t -
CDF p (λ s. real (CS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y × CDF p (λ s. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λ x.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
```

```
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) -
CDF p (λs. real (CS s)) t ×
pos_fn_integral lborel
  (λy.
      f_MA y × CDF p (λs. real (MS s)) y ×
      indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
      pos_fn_integral lborel
        (λx.
            f_cond_MA_MB y x ×
            indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t -
CDF p (λs. real (CS s)) t ×
pos_fn_integral lborel
  (λy.
      f_MA y ×
      (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
       CDF p (λs. real (MS s)) y)) ×
(pos_fn_integral lborel
   (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
       pos_fn_integral lborel
         (λx.
             indicator_fn {w | y < w ∧ w ≤ t} x ×
             f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
       f_P y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t -
CDF p (λs. real (CS s)) t ×
pos_fn_integral lborel
  (λy.
      indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
      pos_fn_integral lborel
        (λx.
            indicator_fn {w | y < w ∧ w ≤ t} x ×
            f_cond_MA_MB y x)) ×
(pos_fn_integral lborel
   (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
       pos_fn_integral lborel
         (λx.
             indicator_fn {w | y < w ∧ w ≤ t} x ×
             f_cond_P_B y x)) +
 pos_fn_integral lborel
```

```
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t -
CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y × CDF p (λ s. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λ x.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) -
CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y × CDF p (λ s. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λ x.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t -
CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λ s. real (MS s)) y)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
```

```
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t -
CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λx.
             indicator_fn {w | y < w ∧ w ≤ t} x ×
             f_cond_MA_MB y x)) ×
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t -
pos_fn_integral lborel
  (λy.
       f_MA y × CDF p (λs. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λx.
             f_cond_MA_MB y x ×
             indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
```

```
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
CDF p (λs. real (CS s)) t × CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
       f_MA y × CDF p (λs. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λx.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λs. real (B_d s)) y))) +
CDF p (λs. real (CS s)) t × CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
       f_MA y × CDF p (λs. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λx.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
CDF p (λs. real (CS s)) t × CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
       f_MA y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λs. real (MS s)) y)) ×
(pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λx.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
```

```
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t +
CDF p (λ s. real (CS s)) t × CDF p (λ s. real (SS s)) t ×
pos_fn_integral lborel
  (λ y.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_MA y ×
       pos_fn_integral lborel
         (λ x.
              indicator_fn {w | y < w ∧ w ≤ t} x ×
              f_cond_MA_MB y x)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
         CDF p (λ s. real (B_d s)) y))) ×
CDF p (λ s. real (PA s)) t × CDF p (λ s. real (PB s)) t ×
CDF p (λ s. real (PS s)) t +
CDF p (λ s. real (CS s)) t ×
pos_fn_integral lborel
  (λ y.
       f_MA y × CDF p (λ s. real (MS s)) y ×
       indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
       pos_fn_integral lborel
         (λ x.
              f_cond_MA_MB y x ×
              indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
(pos_fn_integral lborel
   (λ y.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
        pos_fn_integral lborel
          (λ x.
               indicator_fn {w | y < w ∧ w ≤ t} x ×
               f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λ y.
        f_P y ×
        (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
```

```
        CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t +
CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
      f_MA y × CDF p (λs. real (MS s)) y ×
      indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
      pos_fn_integral lborel
        (λx.
            f_cond_MA_MB y x ×
            indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
(pos_fn_integral lborel
   (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
       pos_fn_integral lborel
         (λx.
             indicator_fn {w | y < w ∧ w ≤ t} x ×
             f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
       f_P y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t -
CDF p (λs. real (CS s)) t × CDF p (λs. real (SS s)) t ×
pos_fn_integral lborel
  (λy.
      f_MA y × CDF p (λs. real (MS s)) y ×
      indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
      pos_fn_integral lborel
        (λx.
            f_cond_MA_MB y x ×
            indicator_fn {x' | y < x' ∧ x' ≤ t} x)) ×
(pos_fn_integral lborel
   (λy.
       indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_P y ×
       pos_fn_integral lborel
         (λx.
             indicator_fn {w | y < w ∧ w ≤ t} x ×
             f_cond_P_B y x)) +
 pos_fn_integral lborel
   (λy.
       f_P y ×
       (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
        CDF p (λs. real (B_d s)) y))) ×
CDF p (λs. real (PA s)) t × CDF p (λs. real (PB s)) t ×
CDF p (λs. real (PS s)) t)
```

[CAS_general_reduced]

```
⊢ ∀PA PB PS MS MA MB B_a B_d CS SS P B.
      (∀s.
           ALL_DISTINCT
             [MA s; MS s; PA s; PB s; PS s; MB s; P s; B_d s;
              B_a s; CS s; SS s]) ∧ (D_BEFORE B_a P = NEVER) ⇒
      (D_OR
         (D_OR
            (D_AND (shared_spare PA PB PS PS)
               (shared_spare PB PA PS PS))
            (D_OR (P_AND MS MA) (CSP MA MB)))
         (WSP (FDEP (D_OR CS SS) P) (FDEP (D_OR CS SS) B_a)
            (FDEP (D_OR CS SS) B_d)) =
       D_OR
         (D_OR
            (D_OR (D_OR CS SS) (D_OR (P_AND MS MA) (CSP MA MB)))
            (WSP P B_a B_d)) (D_AND (D_AND PA PB) PS))
```

[CAS_general_UNION_LIST]

```
⊢ ∀PA PB PS MS MA MB CS SS P B_a B_d p t.
      DFT_event p
        (D_OR CS
           (D_OR SS
              (D_OR (D_AND MA (D_BEFORE MS MA))
                 (D_OR (D_AND MB (D_BEFORE MA MB))
                    (D_OR (D_AND B_a (D_BEFORE P B_a))
                       (D_OR (D_AND P (D_BEFORE B_d P))
                          (D_AND (D_AND PA PB) PS))))))) t =
      union_list
        [DFT_event p CS t; DFT_event p SS t;
         DFT_event p (D_AND MA (D_BEFORE MS MA)) t;
         DFT_event p (D_AND MB (D_BEFORE MA MB)) t;
         DFT_event p
           (D_OR (D_AND B_a (D_BEFORE P B_a))
              (D_AND P (D_BEFORE B_d P))) t;
         DFT_event p (D_AND (D_AND PA PB) PS) t]
```

[CAS_TE1]

```
⊢ ∀T P B_a B_d.
      (D_BEFORE B_a P = NEVER) ⇒
      (WSP (D_OR T P) (D_OR T B_a) (D_OR T B_d) =
       D_OR T (WSP P B_a B_d))
```

[CSP_def2]

```
⊢ ∀X Y. CSP Y X = D_AND X (D_BEFORE Y X)
```

[IN_REST]

```
⊢ ∀x s. x ∈ REST s ⇐⇒ x ∈ s ∧ x ≠ CHOICE s
```

[IN_UNIONL]

```
⊢ ∀l v. v ∈ UNIONL l ⇔ ∃s. MEM s l ∧ v ∈ s
```

[indep_vars_setsg_def]

```
⊢ indep_vars_setsg
    [A0; A1; A2; A3; A4; A5; A6; A7; A8; A9; A10] p t ⇔
  random_variable (λs. real (A9 s)) p borel ∧
  random_variable (λs. real (A5 s)) p borel ∧
  indep_vars p (λi. lborel)
    (λi.
         if i = 0 then (λs. real (A0 s))
         else if i = 1 then (λs. real (A1 s))
         else if i = 2 then (λs. real (A2 s))
         else if i = 3 then (λs. real (A3 s))
         else if i = 4 then (λs. real (A4 s))
         else if i = 5 then (λs. real (A6 s))
         else if i = 6 then (λs. real (A7 s))
         else if i = 7 then (λs. real (A8 s))
         else (λs. real (A10 s))) {0; 1; 2; 3; 4; 5; 6; 7; 8} ∧
  indep_sets p
    (λi.
         {if i = 0 then
            PREIMAGE (λs. real (A0 s)) {u | u ≤ t} ∩ p_space p
          else if i = 1 then
            PREIMAGE (λs. real (A1 s)) {u | u ≤ t} ∩ p_space p
          else if i = 2 then
            PREIMAGE (λs. real (A2 s)) {u | u ≤ t} ∩ p_space p
          else if i = 3 then DFT_event p (CSP A4 A5) t
          else if i = 4 then DFT_event p (WSP A8 A9 A10) t
          else if i = 5 then
            PREIMAGE (λs. real (A6 s)) {u | u ≤ t} ∩ p_space p
          else
            PREIMAGE (λs. real (A7 s)) {u | u ≤ t} ∩ p_space p})
    {0; 1; 2; 3; 4; 5; 6} ∧
  indep_sets p
    (λi.
         {if i = 0 then
            PREIMAGE (λs. real (A0 s)) {u | u ≤ t} ∩ p_space p
          else if i = 1 then
            PREIMAGE (λs. real (A1 s)) {u | u ≤ t} ∩ p_space p
          else if i = 2 then
            PREIMAGE (λs. real (A2 s)) {u | u ≤ t} ∩ p_space p
          else if i = 3 then
            PREIMAGE (λs. (real (A3 s),real (A4 s)))
              {(u,w) | u < w ∧ 0 ≤ w ∧ w ≤ t} ∩ p_space p
          else if i = 4 then DFT_event p (WSP A8 A9 A10) t
          else if i = 5 then
            PREIMAGE (λs. real (A6 s)) {u | u ≤ t} ∩ p_space p
          else
```

```
              PREIMAGE (λs. real (A7 s)) {u | u ≤ t} ∩ p_space p})
        {0; 1; 2; 3; 4; 5; 6} ∧
      indep_sets p
        (λi.
             {if i = 0 then
                PREIMAGE (λs. real (A0 s)) {u | u ≤ t} ∩ p_space p
              else if i = 1 then
                PREIMAGE (λs. real (A1 s)) {u | u ≤ t} ∩ p_space p
              else if i = 2 then
                PREIMAGE (λs. real (A2 s)) {u | u ≤ t} ∩ p_space p
              else if i = 3 then
                PREIMAGE (λs. (real (A3 s),real (A4 s)))
                  {(u,w) | u < w ∧ 0 ≤ w ∧ w ≤ t} ∩ p_space p ∩
                DFT_event p (CSP A4 A5) t
              else if i = 4 then DFT_event p (WSP A8 A9 A10) t
              else if i = 5 then
                PREIMAGE (λs. real (A6 s)) {u | u ≤ t} ∩ p_space p
              else
                PREIMAGE (λs. real (A7 s)) {u | u ≤ t} ∩ p_space p})
        {0; 1; 2; 3; 4; 5; 6}
```

[indep_vars_setsg_ind]

```
 ⊢ ∀P.
       (∀A0 A1 A2 A3 A4 A5 A6 A7 A8 A9 A10 p t.
            P [A0; A1; A2; A3; A4; A5; A6; A7; A8; A9; A10] p t) ∧
       (∀v4. P [] (FST v4) (SND v4)) ∧
       (∀v7 v8. P [v7] (FST v8) (SND v8)) ∧
       (∀v12 v11 v13. P [v12; v11] (FST v13) (SND v13)) ∧
       (∀v18 v17 v16 v19. P [v18; v17; v16] (FST v19) (SND v19)) ∧
       (∀v25 v24 v23 v22 v26.
            P [v25; v24; v23; v22] (FST v26) (SND v26)) ∧
       (∀v33 v32 v31 v30 v29 v34.
            P [v33; v32; v31; v30; v29] (FST v34) (SND v34)) ∧
       (∀v42 v41 v40 v39 v38 v37 v43.
            P [v42; v41; v40; v39; v38; v37] (FST v43) (SND v43)) ∧
       (∀v52 v51 v50 v49 v48 v47 v46 v53.
            P [v52; v51; v50; v49; v48; v47; v46] (FST v53)
              (SND v53)) ∧
       (∀v63 v62 v61 v60 v59 v58 v57 v56 v64.
            P [v63; v62; v61; v60; v59; v58; v57; v56] (FST v64)
              (SND v64)) ∧
       (∀v75 v74 v73 v72 v71 v70 v69 v68 v67 v76.
            P [v75; v74; v73; v72; v71; v70; v69; v68; v67]
              (FST v76) (SND v76)) ∧
       (∀v88 v87 v86 v85 v84 v83 v82 v81 v80 v79 v89.
            P [v88; v87; v86; v85; v84; v83; v82; v81; v80; v79]
              (FST v89) (SND v89)) ∧
       (∀v106 v105 v104 v103 v102 v101 v100 v99 v98 v97 v96 v92
            v93 v107.
```

```
        P
          (v106::v105::v104::v103::v102::v101::v100::v99::
               v98::v97::v96::v92::v93) (FST v107)
          (SND v107)) ⇒
   ∀ v  v1  v2. P v v1 v2
```

[PAND_CSP_event]

```
⊢ ∀ X  Y  Z  p  t.
     rv_gt0_ninfinity [X; Y; Z] ∧
     (∀ s. ALL_DISTINCT [X s; Y s; Z s]) ⇒
     (DFT_event p
        (D_AND (D_AND X (D_BEFORE Z Y)) (D_BEFORE Y X)) t =
      {s |
       0 ≤ Z s ∧ Z s < Y s ∧ 0 ≤ Y s ∧ Y s ≤ X s ∧
       X s ≤ Normal t} ∩ p_space p)
```