Contents

1 drive_by_wire Theory

Built: 17 December 2018
Parent Theories: AND_FDEP

1.1 Definitions

[indep_vars_7_def]

```
⊢ ∀p M A0 A1 A2 A3 A4 A5 A6.
      indep_vars_7 p M A0 A1 A2 A3 A4 A5 A6 ⟺
      indep_vars p (λi. M)
        (λi.
             if i = 0 then A0
             else if i = 1 then A1
             else if i = 2 then A2
             else if i = 3 then A3
             else if i = 4 then A4
             else if i = 5 then A5
             else A6) {0; 1; 2; 3; 4; 5; 6}
```

[UNIONL_def]

```
⊢ (UNIONL [] = {}) ∧ ∀s ss. UNIONL (s::ss) = s ∪ UNIONL ss
```

1.2 Theorems

[DFT_event_IN_events_p]

```
⊢ ∀X p t.
      random_variable (λx. real (X x)) p borel ∧
      (∀s. 0 ≤ X s ∧ X s ≠ PosInf) ∧ 0 ≤ t ⇒
      DFT_event p X t ∈ events p
```

[drive_by_wire_top_event]

```
⊢ ∀BS TS PCCU SCCU_a SCCU_d BCU EF TF.
      (D_AND SCCU_a SCCU_d = NEVER) ∧
      (∀s.
           ALL_DISTINCT
             [BS s; TS s; PCCU s; SCCU_a s; SCCU_d s; BCU s;
              EF s; TF s]) ⇒
      (D_OR
         (D_OR (D_OR (D_OR TF EF) (WSP PCCU SCCU_a SCCU_d)) BCU)
         (D_OR TS BS) =
       D_OR
         (D_OR
            (D_OR
               (D_OR (D_OR (D_OR TF EF) BCU)
                  (D_AND SCCU_a (D_BEFORE PCCU SCCU_a)))
               (D_AND PCCU (D_BEFORE SCCU_d PCCU))) TS) BS)
```

[drive_by_wire_union_list]

$\vdash$ $\forall$ *BS TS PCCU SCCU_a SCCU_d BCU SCU EF TF p t*.
    DFT_event *p*
      (D_OR
        (D_OR (D_OR (D_OR *TF EF*) (WSP *PCCU SCCU_a SCCU_d*))
          BCU) (D_OR *TS BS*)) *t* =
    union_list
      [DFT_event *p TF t*; DFT_event *p EF t*; DFT_event *p BCU t*;
       DFT_event *p* (WSP *PCCU SCCU_a SCCU_d*) *t*;
       DFT_event *p BS t*; DFT_event *p TS t*]

[IN_REST]

$\vdash \forall x\ s.\ x \in$ REST $s \iff x \in s \land x \neq$ CHOICE s

[IN_UNIONL]

$\vdash \forall l\ v.\ v \in$ UNIONL $l \iff \exists s.$ MEM $s\ l \land v \in s$

[indep_vars_sets_drive_def]

$\vdash$ indep_vars_sets_drive $[A_0;\ A_1;\ A_2;\ A_3;\ A_4;\ A_5;\ A_6;\ A_7]\ p\ t \iff$
  indep_vars_7 p lborel $(\lambda s.$ real $(A_0\ s))$ $(\lambda s.$ real $(A_1\ s))$
    $(\lambda s.$ real $(A_2\ s))$ $(\lambda s.$ real $(A_4\ s))$ $(\lambda s.$ real $(A_5\ s))$
    $(\lambda s.$ real $(A_6\ s))$ $(\lambda s.$ real $(A_7\ s)) \land$
  indep_sets p
    $(\lambda i.$
        {**if** $i = 0$ **then**
           PREIMAGE $(\lambda s.$ real $(A_0\ s))$ $\{u \mid u \le t\} \cap$ p_space p
         else if $i = 1$ **then**
           PREIMAGE $(\lambda s.$ real $(A_1\ s))$ $\{u \mid u \le t\} \cap$ p_space p
         else if $i = 2$ **then**
           PREIMAGE
             $(\lambda s.$ (real $(A_2\ s)$,real $(A_3\ s)$,real $(A_4\ s)))
             $\{(y,xa,xd) \mid$
             $y < xa \land xa \le t \land 0 \le y \land y \le t \lor xd < y \land y \le t\} \cap$
           p_space p
         else if $i = 3$ **then**
           PREIMAGE $(\lambda s.$ real $(A_5\ s))$ $\{u \mid u \le t\} \cap$ p_space p
         else if $i = 4$ **then**
           PREIMAGE $(\lambda s.$ real $(A_6\ s))$ $\{u \mid u \le t\} \cap$ p_space p
         else
           PREIMAGE $(\lambda s.$ real $(A_7\ s))$ $\{u \mid u \le t\} \cap$ p_space p})
    $\{0;\ 1;\ 2;\ 3;\ 4;\ 5\}$

[indep_vars_sets_drive_ind]

$\vdash \forall P.$
    $(\forall A_0\ A_1\ A_2\ A_3\ A_4\ A_5\ A_6\ A_7\ p\ t.$
        $P\ [A_0;\ A_1;\ A_2;\ A_3;\ A_4;\ A_5;\ A_6;\ A_7]\ p\ t) \land$
    $(\forall v_4.\ P$ [] (FST v_4) (SND v_4)) $\land$
    $(\forall v_7\ v_8.\ P\ [v_7]$ (FST v_8) (SND v_8)) $\land$

$(\forall v_{12}\ v_{11}\ v_{13}.\ P\ [v_{12};\ v_{11}]\ (\texttt{FST}\ v_{13})\ (\texttt{SND}\ v_{13}))\ \land$
$(\forall v_{18}\ v_{17}\ v_{16}\ v_{19}.\ P\ [v_{18};\ v_{17};\ v_{16}]\ (\texttt{FST}\ v_{19})\ (\texttt{SND}\ v_{19}))\ \land$
$(\forall v_{25}\ v_{24}\ v_{23}\ v_{22}\ v_{26}.$
$\quad P\ [v_{25};\ v_{24};\ v_{23};\ v_{22}]\ (\texttt{FST}\ v_{26})\ (\texttt{SND}\ v_{26}))\ \land$
$(\forall v_{33}\ v_{32}\ v_{31}\ v_{30}\ v_{29}\ v_{34}.$
$\quad P\ [v_{33};\ v_{32};\ v_{31};\ v_{30};\ v_{29}]\ (\texttt{FST}\ v_{34})\ (\texttt{SND}\ v_{34}))\ \land$
$(\forall v_{42}\ v_{41}\ v_{40}\ v_{39}\ v_{38}\ v_{37}\ v_{43}.$
$\quad P\ [v_{42};\ v_{41};\ v_{40};\ v_{39};\ v_{38};\ v_{37}]\ (\texttt{FST}\ v_{43})\ (\texttt{SND}\ v_{43}))\ \land$
$(\forall v_{52}\ v_{51}\ v_{50}\ v_{49}\ v_{48}\ v_{47}\ v_{46}\ v_{53}.$
$\quad P\ [v_{52};\ v_{51};\ v_{50};\ v_{49};\ v_{48};\ v_{47};\ v_{46}]\ (\texttt{FST}\ v_{53})$
$\quad\ (\texttt{SND}\ v_{53}))\ \land$
$(\forall v_{67}\ v_{66}\ v_{65}\ v_{64}\ v_{63}\ v_{62}\ v_{61}\ v_{60}\ v_{56}\ v_{57}\ v_{68}.$
$\quad P\ (v_{67}::v_{66}::v_{65}::v_{64}::v_{63}::v_{62}::v_{61}::v_{60}::v_{56}::v_{57})$
$\quad\ (\texttt{FST}\ v_{68})\ (\texttt{SND}\ v_{68}))\ \Rightarrow$
$\forall v\ v_1\ v_2.\ P\ v\ v_1\ v_2$

[normal_real_mul]

$\vdash \forall x\ y.$
$\quad x \neq \texttt{PosInf} \land x \neq \texttt{NegInf} \land y \neq \texttt{PosInf} \land y \neq \texttt{NegInf} \Rightarrow$
$\quad (\texttt{Normal}\ (\texttt{real}\ x \times \texttt{real}\ y) =$
$\quad\ \texttt{Normal}\ (\texttt{real}\ x) \times \texttt{Normal}\ (\texttt{real}\ y))$

[PREIMAGE_EXTREAL_REAL_3RV_WSP]

$\vdash \forall Y\ X\_a\ X\_d\ p\ t.$
$\quad (\forall s.$
$\qquad X\_a\ s \neq \texttt{PosInf} \land 0 \leq X\_a\ s \land 0 \leq X\_d\ s \land$
$\qquad X\_d\ s \neq \texttt{PosInf} \land Y\ s \neq \texttt{PosInf} \land 0 \leq Y\ s) \land 0 \leq t \Rightarrow$
$\quad (\texttt{PREIMAGE}\ (\lambda x.\ (Y\ x,X\_a\ x,X\_d\ x))$
$\qquad \{(y',xa',xd')\ |$
$\qquad y' < xa' \land xa' \leq \texttt{Normal}\ t \land 0 \leq y' \land y' \leq \texttt{Normal}\ t \lor$
$\qquad xd' < y' \land y' \leq \texttt{Normal}\ t\} \cap \texttt{p\_space}\ p =$
$\quad\ \texttt{PREIMAGE}\ (\lambda x.\ (\texttt{real}\ (Y\ x),\texttt{real}\ (X\_a\ x),\texttt{real}\ (X\_d\ x)))$
$\qquad \{(y,xa,xd)\ |$
$\qquad y < xa \land xa \leq t \land 0 \leq y \land y \leq t \lor xd < y \land y \leq t\} \cap$
$\quad\ \texttt{p\_space}\ p)$

[prob_drive_by_wire_initial2]

$\vdash \forall BS\ TS\ PCCU\ SCCU\_a\ SCCU\_d\ BCU\ EF\ TF\ p\ t\ f\_pccu\ f\_cond$
$\quad f\_sccu\_pccu.$
$\quad (\forall s.$
$\qquad \texttt{ALL\_DISTINCT}$
$\qquad\ [BS\ s;\ TS\ s;\ PCCU\ s;\ SCCU\_a\ s;\ SCCU\_d\ s;\ BCU\ s;$
$\qquad\ \ EF\ s;\ TF\ s]) \land$
$\quad \texttt{All\_distinct\_events}\ p$
$\quad\ [TF;\ EF;\ BCU;\ \texttt{WSP}\ PCCU\ SCCU\_a\ SCCU\_d;\ BS;\ TS]\ t \land$
$\quad 0 \leq t \land$
$\quad \texttt{rv\_gt0\_ninfinity}$
$\quad\ [BS;\ TS;\ PCCU;\ SCCU\_a;\ SCCU\_d;\ BCU;\ EF;\ TF] \land$

```
random_variable (λ x. real (SCCU_a x)) p borel ∧
(D_AND SCCU_a SCCU_d = NEVER) ∧
(∀ y.
     cond_density lborel lborel p (λ s. real (SCCU_a s))
       (λ s. real (PCCU s)) y f_sccu_pccu f_pccu f_cond) ∧
den_gt0_ninfinity f_sccu_pccu f_pccu f_cond ∧
indep_var p lborel (λ s. real (SCCU_d s)) lborel
  (λ s. real (PCCU s)) ∧
cont_CDF p (λ s. real (SCCU_d s)) ∧
measurable_CDF p (λ s. real (SCCU_d s)) ∧
indep_vars_sets_drive
  [BS; TS; PCCU; SCCU_a; SCCU_d; BCU; EF; TF] p t ⇒
(prob p
   (DFT_event p
      (D_OR
         (D_OR
            (D_OR (D_OR TF EF) (WSP PCCU SCCU_a SCCU_d))
            BCU) (D_OR TS BS)) t) =
 CDF p (λ s. real (TF s)) t + CDF p (λ s. real (EF s)) t +
 CDF p (λ s. real (BCU s)) t +
 (pos_fn_integral lborel
    (λ pccu.
         indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × f_pccu pccu ×
         pos_fn_integral lborel
           (λ sccu_a.
                indicator_fn
                  {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                  sccu_a × f_cond pccu sccu_a)) +
  pos_fn_integral lborel
    (λ pccu.
         f_pccu pccu ×
         (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
            pccu × CDF p (λ s. real (SCCU_d s)) pccu))) +
 CDF p (λ s. real (BS s)) t + CDF p (λ s. real (TS s)) t -
 CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t -
 CDF p (λ s. real (TF s)) t × CDF p (λ s. real (BCU s)) t -
 CDF p (λ s. real (TF s)) t ×
 (pos_fn_integral lborel
    (λ pccu.
         indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × f_pccu pccu ×
         pos_fn_integral lborel
           (λ sccu_a.
                indicator_fn
                  {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                  sccu_a × f_cond pccu sccu_a)) +
  pos_fn_integral lborel
    (λ pccu.
```

```
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λs. real (SCCU_d s)) pccu))) -
CDF p (λs. real (TF s)) t × CDF p (λs. real (BS s)) t -
CDF p (λs. real (TF s)) t × CDF p (λs. real (TS s)) t -
CDF p (λs. real (EF s)) t × CDF p (λs. real (BCU s)) t -
CDF p (λs. real (EF s)) t ×
(pos_fn_integral lborel
   (λpccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λsccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λpccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λs. real (SCCU_d s)) pccu))) -
CDF p (λs. real (EF s)) t × CDF p (λs. real (BS s)) t -
CDF p (λs. real (EF s)) t × CDF p (λs. real (TS s)) t -
CDF p (λs. real (BCU s)) t ×
(pos_fn_integral lborel
   (λpccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λsccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λpccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λs. real (SCCU_d s)) pccu))) -
CDF p (λs. real (BCU s)) t × CDF p (λs. real (BS s)) t -
CDF p (λs. real (BCU s)) t × CDF p (λs. real (TS s)) t -
(pos_fn_integral lborel
   (λpccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λsccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
```

```
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t -
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
CDF p (λ s. real (BCU s)) t +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
CDF p (λ s. real (BS s)) t +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
CDF p (λ s. real (TS s)) t +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
```

```
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (BCU s)) t ×
CDF p (λ s. real (BS s)) t +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (BCU s)) t ×
CDF p (λ s. real (TS s)) t +
CDF p (λ s. real (TF s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t +
CDF p (λ s. real (TF s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (TS s)) t +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (BS s)) t ×
CDF p (λ s. real (TS s)) t +
CDF p (λ s. real (EF s)) t × CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
```

```
     (λ pccu.
          indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
            pccu × f_pccu pccu ×
          pos_fn_integral lborel
            (λ sccu_a.
                 indicator_fn
                   {sccu_a′ | pccu < sccu_a′ ∧ sccu_a′ ≤ t}
                   sccu_a × f_cond pccu sccu_a)) +
  pos_fn_integral lborel
     (λ pccu.
          f_pccu pccu ×
          (indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
             pccu × CDF p (λ s. real (SCCU_d s)) pccu))) +
 CDF p (λ s. real (EF s)) t × CDF p (λ s. real (BCU s)) t ×
 CDF p (λ s. real (BS s)) t +
 CDF p (λ s. real (EF s)) t × CDF p (λ s. real (BCU s)) t ×
 CDF p (λ s. real (TS s)) t +
 CDF p (λ s. real (EF s)) t ×
 (pos_fn_integral lborel
     (λ pccu.
          indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
            pccu × f_pccu pccu ×
          pos_fn_integral lborel
            (λ sccu_a.
                 indicator_fn
                   {sccu_a′ | pccu < sccu_a′ ∧ sccu_a′ ≤ t}
                   sccu_a × f_cond pccu sccu_a)) +
  pos_fn_integral lborel
     (λ pccu.
          f_pccu pccu ×
          (indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
             pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
 CDF p (λ s. real (BS s)) t +
 CDF p (λ s. real (EF s)) t ×
 (pos_fn_integral lborel
     (λ pccu.
          indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
            pccu × f_pccu pccu ×
          pos_fn_integral lborel
            (λ sccu_a.
                 indicator_fn
                   {sccu_a′ | pccu < sccu_a′ ∧ sccu_a′ ≤ t}
                   sccu_a × f_cond pccu sccu_a)) +
  pos_fn_integral lborel
     (λ pccu.
          f_pccu pccu ×
          (indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
             pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
 CDF p (λ s. real (TS s)) t +
```

```
CDF p (λs. real (EF s)) t × CDF p (λs. real (BS s)) t ×
CDF p (λs. real (TS s)) t +
CDF p (λs. real (BCU s)) t ×
(pos_fn_integral lborel
   (λpccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λsccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λpccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λs. real (SCCU_d s)) pccu))) ×
CDF p (λs. real (BS s)) t +
CDF p (λs. real (BCU s)) t ×
(pos_fn_integral lborel
   (λpccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λsccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λpccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λs. real (SCCU_d s)) pccu))) ×
CDF p (λs. real (TS s)) t +
CDF p (λs. real (BCU s)) t × CDF p (λs. real (BS s)) t ×
CDF p (λs. real (TS s)) t +
(pos_fn_integral lborel
   (λpccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λsccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λpccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
```

```
          pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × CDF p (λ s. real (SCCU_d s)) pccu))) -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
CDF p (λ s. real (BCU s)) t × CDF p (λ s. real (BS s)) t -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
CDF p (λ s. real (BCU s)) t × CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
```

```
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (BCU s)) t ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (TF s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
```

```
                  {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                  sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (EF s)) t × CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t -
CDF p (λ s. real (EF s)) t × CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (EF s)) t × CDF p (λ s. real (BCU s)) t ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (EF s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
```

```
                indicator_fn
                  {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                  sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
                indicator_fn
                  {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                  sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t +
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t) +
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p TS t) +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
CDF p (λ s. real (BCU s)) t × CDF p (λ s. real (BS s)) t ×
CDF p (λ s. real (TS s)) t +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
                indicator_fn
                  {sccu_a' | pccu < sccu_a' ∧ sccu_a' ≤ t}
                  sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
```

```
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t +
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a′ | pccu < sccu_a′ ∧ sccu_a′ ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t +
CDF p (λ s. real (EF s)) t × CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a′ | pccu < sccu_a′ ∧ sccu_a′ ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
   (λ pccu.
        f_pccu pccu ×
        (indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
           pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t -
CDF p (λ s. real (TF s)) t × CDF p (λ s. real (EF s)) t ×
CDF p (λ s. real (BCU s)) t ×
(pos_fn_integral lborel
   (λ pccu.
        indicator_fn {pccu′ | 0 ≤ pccu′ ∧ pccu′ ≤ t}
          pccu × f_pccu pccu ×
        pos_fn_integral lborel
          (λ sccu_a.
               indicator_fn
                 {sccu_a′ | pccu < sccu_a′ ∧ sccu_a′ ≤ t}
                 sccu_a × f_cond pccu sccu_a)) +
 pos_fn_integral lborel
```

```
      (λ pccu.
           f_pccu pccu ×
           (indicator_fn {pccu' | 0 ≤ pccu' ∧ pccu' ≤ t}
              pccu × CDF p (λ s. real (SCCU_d s)) pccu))) ×
    CDF p (λ s. real (BS s)) t × CDF p (λ s. real (TS s)) t)
```

[prob_drive_by_wire_PIE]

```
⊢ ∀ BS TS PCCU SCCU_a SCCU_d BCU EF TF p t.
    (∀ s.
         ALL_DISTINCT
           [BS s; TS s; PCCU s; SCCU_a s; SCCU_d s; BCU s;
            EF s; TF s]) ∧
    All_distinct_events p
      [TF; EF; BCU; WSP PCCU SCCU_a SCCU_d; BS; TS] t ∧
    0 ≤ t ∧
    rv_gt0_ninfinity
      [BS; TS; PCCU; SCCU_a; SCCU_d; BCU; EF; TF] ∧
    random_variable (λ x. real (TF x)) p borel ∧
    random_variable (λ x. real (EF x)) p borel ∧
    random_variable (λ x. real (BCU x)) p borel ∧
    random_variable (λ x. real (SCCU_d x)) p borel ∧
    random_variable (λ x. real (SCCU_a x)) p borel ∧
    random_variable (λ x. real (PCCU x)) p borel ∧
    random_variable (λ x. real (TS x)) p borel ∧
    random_variable (λ x. real (BS x)) p borel ⇒
    (prob p
       (DFT_event p
          (D_OR
             (D_OR
                (D_OR (D_OR TF EF) (WSP PCCU SCCU_a SCCU_d))
                BCU) (D_OR TS BS)) t) =
     prob p (DFT_event p TF t) + prob p (DFT_event p EF t) +
     prob p (DFT_event p BCU t) +
     prob p (DFT_event p (WSP PCCU SCCU_a SCCU_d) t) +
     prob p (DFT_event p BS t) + prob p (DFT_event p TS t) -
     prob p (DFT_event p TF t ∩ DFT_event p EF t) -
     prob p (DFT_event p TF t ∩ DFT_event p BCU t) -
     prob p
       (DFT_event p TF t ∩
        DFT_event p (WSP PCCU SCCU_a SCCU_d) t) -
     prob p (DFT_event p TF t ∩ DFT_event p BS t) -
     prob p (DFT_event p TF t ∩ DFT_event p TS t) -
     prob p (DFT_event p EF t ∩ DFT_event p BCU t) -
     prob p
       (DFT_event p EF t ∩
        DFT_event p (WSP PCCU SCCU_a SCCU_d) t) -
     prob p (DFT_event p EF t ∩ DFT_event p BS t) -
     prob p (DFT_event p EF t ∩ DFT_event p TS t) -
     prob p
```

```
  (DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t) -
prob p (DFT_event p BCU t ∩ DFT_event p BS t) -
prob p (DFT_event p BCU t ∩ DFT_event p TS t) -
prob p
  (DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t) -
prob p
  (DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p TS t) -
prob p (DFT_event p BS t ∩ DFT_event p TS t) +
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BCU t) +
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t) +
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BS t) +
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p TF t ∩ DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t) +
prob p
  (DFT_event p TF t ∩ DFT_event p BCU t ∩
   DFT_event p BS t) +
prob p
  (DFT_event p TF t ∩ DFT_event p BCU t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p TF t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t) +
prob p
  (DFT_event p TF t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p TF t ∩ DFT_event p BS t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p EF t ∩ DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t) +
prob p
  (DFT_event p EF t ∩ DFT_event p BCU t ∩
   DFT_event p BS t) +
```

```
prob p
  (DFT_event p EF t ∩ DFT_event p BCU t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p EF t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t) +
prob p
  (DFT_event p EF t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p EF t ∩ DFT_event p BS t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t) +
prob p
  (DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p BCU t ∩ DFT_event p BS t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t ∩ DFT_event p TS t) -
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t) -
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BCU t ∩ DFT_event p BS t) -
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BCU t ∩ DFT_event p TS t) -
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t) -
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p TS t) -
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BS t ∩ DFT_event p TS t) -
```

```
prob p
  (DFT_event p TF t ∩ DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t) -
prob p
  (DFT_event p TF t ∩ DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p TS t) -
prob p
  (DFT_event p TF t ∩ DFT_event p BCU t ∩
   DFT_event p BS t ∩ DFT_event p TS t) -
prob p
  (DFT_event p TF t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t ∩ DFT_event p TS t) -
prob p
  (DFT_event p EF t ∩ DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t) -
prob p
  (DFT_event p EF t ∩ DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p TS t) -
prob p
  (DFT_event p EF t ∩ DFT_event p BCU t ∩
   DFT_event p BS t ∩ DFT_event p TS t) -
prob p
  (DFT_event p EF t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t ∩ DFT_event p TS t) -
prob p
  (DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t ∩ DFT_event p TS t) +
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p BS t) +
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BCU t ∩
   DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
   DFT_event p TS t) +
prob p
  (DFT_event p TF t ∩ DFT_event p EF t ∩
   DFT_event p BCU t ∩ DFT_event p BS t ∩
   DFT_event p TS t) +
prob p
```

```
      (DFT_event p TF t ∩ DFT_event p EF t ∩
       DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
       DFT_event p BS t ∩ DFT_event p TS t) +
    prob p
      (DFT_event p TF t ∩ DFT_event p BCU t ∩
       DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
       DFT_event p BS t ∩ DFT_event p TS t) +
    prob p
      (DFT_event p EF t ∩ DFT_event p BCU t ∩
       DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
       DFT_event p BS t ∩ DFT_event p TS t) -
    prob p
      (DFT_event p TF t ∩ DFT_event p EF t ∩
       DFT_event p BCU t ∩
       DFT_event p (WSP PCCU SCCU_a SCCU_d) t ∩
       DFT_event p BS t ∩ DFT_event p TS t))
```

[WSP_PREIMAGE]

```
 ⊢ ∀ Y X_a X_d p t.
       {s |
        Y s < X_a s ∧ X_a s ≤ Normal t ∧ 0 ≤ Y s ∧ Y s ≤ Normal t} ∩
       p_space p ∪
       {s | X_d s < Y s ∧ Y s ≤ Normal t} ∩ p_space p =
       PREIMAGE (λx. (Y x,X_a x,X_d x))
         {(y,xa,xd) |
          y < xa ∧ xa ≤ Normal t ∧ 0 ≤ y ∧ y ≤ Normal t ∨
          xd < y ∧ y ≤ Normal t} ∩ p_space p
```