Contents

1 Q1_shared_spare Theory

Built: 08 July 2019
Parent Theories: CPDFT, AND_FDEP

1.1 Definitions

[UNIONL_def]
$\vdash$ (UNIONL [] = { }) $\wedge$ $\forall s\ ss$. UNIONL (s::ss) = s $\cup$ UNIONL ss

1.2 Theorems

[add_assoc_6_var]
$\vdash \forall A_1\ A_2\ A_3\ A_4\ A_5\ A_6.$
$A_1 \neq$ PosInf $\wedge$ $A_2 \neq$ PosInf $\wedge$ $A_3 \neq$ PosInf $\wedge$ $A_4 \neq$ PosInf $\wedge$
$A_5 \neq$ PosInf $\wedge$ $A_6 \neq$ PosInf $\Rightarrow$
$(A_1 + (A_2 + (A_3 + (A_4 + (A_5 + A_6))))) =$
$A_1 + A_2 + A_3 + A_4 + A_5 + A_6)$

[after_set1_BIGUNION_IN_MESURABLE_SETS]
$\vdash \forall t\ q.$
{w | real $q < w \wedge 0 \leq w \wedge w \leq t$} $\times$ {u | $u <$ real q} $\in$
measurable_sets (pair_measure lborel lborel)

[after_set1_BIGUNION_Q]
$\vdash \forall t.$
BIGUNION
{{w | real $q < w \wedge 0 \leq w \wedge w \leq t$} $\times$ {u | $u <$ real q} |
$q \in$ Q_set} =
{(a,u) | $u < a \wedge 0 \leq a \wedge a \leq t$}

[after_set1_IN_MEASURABLE_SETS_PAIR_lborel]
$\vdash \forall t.$
{(a,u) | $u < a \wedge 0 \leq a \wedge a \leq t$} $\in$
measurable_sets (pair_measure lborel lborel)

[DFT_event_after1_PREIMAGE]
$\vdash \forall p\ t\ X\ Y.$
DFT_event p (D_AND Y (D_BEFORE X Y)) t =
PREIMAGE (λx. (Y x,X x)) {(w,u) | $u < w \wedge w \leq$ Normal t} $\cap$
p_space p

[DFT_event_after1_PREIMAGE_GT0]
$\vdash \forall p\ t\ X\ Y.$
($\forall s.\ 0 \leq Y\ s$) $\Rightarrow$
(DFT_event p (D_AND Y (D_BEFORE X Y)) t =
PREIMAGE (λx. (Y x,X x))
{(w,u) | $u < w \wedge 0 \leq w \wedge w \leq$ Normal t} $\cap$ p_space p)

[DFT_event_after1_PREIMAGE_GT0_REAL]

```
⊢ ∀p t X Y.
      rv_gt0_ninfinity [X; Y] ∧ 0 ≤ t ⇒
      (DFT_event p (D_AND Y (D_BEFORE X Y)) t =
       PREIMAGE (λx. (real (Y x),real (X x)))
         {(w,u) | u < w ∧ 0 ≤ w ∧ w ≤ t} ∩ p_space p)
```

[IN_REST]

```
⊢ ∀x s. x ∈ REST s ⇔ x ∈ s ∧ x ≠ CHOICE s
```

[IN_UNIONL]

```
⊢ ∀l v. v ∈ UNIONL l ⇔ ∃s. MEM s l ∧ v ∈ s
```

[PREIMAGE_EXTREAL_REAL_2RV_after1]

```
⊢ ∀X Y t p.
      (∀s. X s ≠ PosInf ∧ 0 ≤ X s ∧ Y s ≠ PosInf ∧ 0 ≤ Y s) ∧
      0 ≤ t ⇒
      (PREIMAGE (λs. (Y s,X s))
         {(w,u) | u < w ∧ 0 ≤ w ∧ w ≤ Normal t} ∩ p_space p =
       PREIMAGE (λs. (real (Y s),real (X s)))
         {(w,u) | u < w ∧ 0 ≤ w ∧ w ≤ t} ∩ p_space p)
```

[prob_after_diff_space]

```
⊢ ∀p Xa Y Z t.
      rv_gt0_ninfinity [Xa; Y; Z] ∧ 0 ≤ t ∧
      indep_varp p (pair_measure lborel lborel)
        (λx. (real (Xa x),real (Y x))) lborel (λx. real (Z x)) ⇒
      (prob p
         (DFT_event p (D_AND Xa (D_BEFORE Y Xa)) t ∩
          (p_space p DIFF DFT_event p Z t)) =
       (1 - prob p (DFT_event p Z t)) ×
       prob p (DFT_event p (D_AND Xa (D_BEFORE Y Xa)) t))
```

[prob_shared_spare_initial_1]

```
⊢ ∀Xa Xd Y Z p t.
      rv_gt0_ninfinity [Xa; Xd; Y; Z] ∧
      (∀s. ALL_DISTINCT [Y s; Xa s; Xd s; Z s]) ∧
      DISJOINT_WSP Y Xa Xd t ∧ DISJOINT_WSP Z Xa Xd t ∧
      (∀s. D_AND (D_BEFORE Z Xd) (D_BEFORE Xd Y) s = NEVER s) ∧
      (∀s. D_AND (D_BEFORE Y Xd) (D_BEFORE Xd Z) s = NEVER s) ∧
      (∀s. D_AND (D_BEFORE Xa Y) (D_BEFORE Xa Z) s = NEVER s) ∧
      random_variable (λx. real (Xa x)) p
        (m_space lborel,measurable_sets lborel) ∧
      random_variable (λx. real (Xd x)) p
        (m_space lborel,measurable_sets lborel) ∧
      random_variable (λx. real (Y x)) p
        (m_space lborel,measurable_sets lborel) ∧
```

```
random_variable (λx. real (Z x)) p
  (m_space lborel,measurable_sets lborel) ∧ 0 ≤ t ⇒
(prob p
   (DFT_event p
      (D_OR
         (D_OR (D_AND Xa (D_BEFORE Y Xa))
            (D_AND Y (D_BEFORE Xd Y)))
         (D_AND Y (D_BEFORE Z Y))) t) =
 prob p
   (DFT_event p
      (D_AND (D_AND Xa (D_BEFORE Y Z)) (D_BEFORE Z Xa)) t) +
 prob p
   (DFT_event p
      (D_AND (D_AND Z (D_BEFORE Y Xa)) (D_BEFORE Xa Z)) t) +
 prob p
   (DFT_event p (D_AND Xa (D_BEFORE Y Xa)) t ∩
    (p_space p DIFF DFT_event p Z t)) +
 prob p
   (DFT_event p
      (D_AND (D_AND Z (D_BEFORE Xd Y)) (D_BEFORE Y Z)) t) +
 prob p
   (DFT_event p (D_AND Y (D_BEFORE Xd Y)) t ∩
    (p_space p DIFF DFT_event p Z t)) +
 prob p (DFT_event p (D_AND Y (D_BEFORE Z Y)) t))
```

[prob_shared_spare_initial_2]

```
⊢ ∀p t Xa Xd Y Z f_xay f_condxa_y f_y f_z.
     0 ≤ t ∧
     indep_varp p lborel (λx. real (Z x))
       (pair_measure lborel lborel)
       (λx. (real (Y x),real (Xd x))) ∧
     indep_var p lborel (λx. real (Y x)) lborel
       (λx. real (Xd x)) ∧
     distributed p lborel (λx. real (Z x)) f_z ∧
     (∀z. 0 ≤ f_z z) ∧
     distributed p lborel (λx. real (Y x)) f_y ∧
     (∀x. 0 ≤ f_y x) ∧ cont_CDF p (λs. real (Xd s)) ∧
     measurable_CDF p (λs. real (Xd s)) ∧
     (∀x. (λs. real (CDF p (λx. real (Xd x)) s)) contl x) ∧
     indep_var p lborel (λs. real (Z s)) lborel
       (λs. real (Y s)) ∧ cont_CDF p (λs. real (Z s)) ∧
     measurable_CDF p (λs. real (Z s)) ∧
     (∀s. ALL_DISTINCT [Xa s; Xd s; Y s; Z s]) ∧
     prob_space p ∧ den_gt0_ninfinity f_xay f_y f_condxa_y ∧
     rv_gt0_ninfinity [Xa; Xd; Y; Z] ∧
     (∀y.
          cond_density lborel lborel p (λx. real (Xa x))
            (λx. real (Y x)) y f_xay f_y f_condxa_y) ∧
     indep_varp p (pair_measure lborel lborel)
```

```
  (λx. (real (Xa x),real (Y x))) lborel (λx. real (Z x)) ∧
(∀z. 0 ≤ f_z z) ∧ (∀x. f_xay x ≠ PosInf) ∧
distributed p lborel (λx. real (Z x)) f_z ⇒
(prob p
   (DFT_event p
      (D_AND (D_AND Xa (D_BEFORE Y Z)) (D_BEFORE Z Xa)) t) +
 prob p
   (DFT_event p
      (D_AND (D_AND Z (D_BEFORE Y Xa)) (D_BEFORE Xa Z)) t) +
 prob p
   (DFT_event p (D_AND Xa (D_BEFORE Y Xa)) t ∩
    (p_space p DIFF DFT_event p Z t)) +
 prob p
   (DFT_event p
      (D_AND (D_AND Z (D_BEFORE Xd Y)) (D_BEFORE Y Z)) t) +
 prob p
   (DFT_event p (D_AND Y (D_BEFORE Xd Y)) t ∩
    (p_space p DIFF DFT_event p Z t)) +
 prob p (DFT_event p (D_AND Y (D_BEFORE Z Y)) t) =
 pos_fn_integral lborel
   (λy.
        f_y y × indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
        pos_fn_integral lborel
          (λx.
               f_condxa_y y x ×
               indicator_fn {x' | y < x' ∧ x' ≤ t} x ×
               pos_fn_integral lborel
                 (λz.
                      f_z z ×
                      indicator_fn {z' | y < z' ∧ z' < x}
                        z))) +
 pos_fn_integral lborel
   (λz.
        f_z z × indicator_fn {z' | 0 ≤ z' ∧ z' ≤ t} z ×
        pos_fn_integral lborel
          (λy.
               f_y y ×
               indicator_fn {y' | 0 ≤ y' ∧ y' < z} y ×
               pos_fn_integral lborel
                 (λx.
                      f_condxa_y y x ×
                      indicator_fn {x' | y < x' ∧ x' < z}
                        x))) +
 (1 - CDF p (λx. real (Z x)) t) ×
 pos_fn_integral lborel
   (λy.
        indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_y y ×
        pos_fn_integral lborel
          (λx.
```

```
                  indicator_fn {w | y < w ∧ w ≤ t} x ×
                  f_condxa_y y x)) +
      pos_fn_integral lborel
        (λz.
             f_z z × indicator_fn {z' | 0 ≤ z' ∧ z' ≤ t} z ×
             pos_fn_integral lborel
               (λy.
                    f_y y ×
                    indicator_fn {y' | 0 ≤ y' ∧ y' < z} y ×
                    CDF p (λx. real (Xd x)) y)) +
      (1 - CDF p (λx. real (Z x)) t) ×
      pos_fn_integral lborel
        (λy.
             f_y y ×
             (indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
              CDF p (λs. real (Xd s)) y)) +
      pos_fn_integral lborel
        (λy.
             f_y y ×
             (indicator_fn {y' | 0 ≤ y' ∧ y' ≤ t} y ×
              CDF p (λs. real (Z s)) y)))
```

[shared_spare_disjoint_1]

```
 ⊢ ∀Xa Xd Y Z p t.
       rv_gt0_ninfinity [Xa; Xd; Y; Z] ∧
       (∀s. ALL_DISTINCT [Y s; Xa s; Xd s; Z s]) ∧
       DISJOINT_WSP Y Xa Xd t ∧ DISJOINT_WSP Z Xa Xd t ∧
       (∀s. D_AND (D_BEFORE Z Xd) (D_BEFORE Xd Y) s = NEVER s) ∧
       (∀s. D_AND (D_BEFORE Y Xd) (D_BEFORE Xd Z) s = NEVER s) ∧
       (∀s. D_AND (D_BEFORE Xa Y) (D_BEFORE Xa Z) s = NEVER s) ⇒
       DISJOINT
         (DFT_event p
            (D_AND (D_AND Xa (D_BEFORE Y Z)) (D_BEFORE Z Xa)) t)
         (DFT_event p
            (D_AND (D_AND Z (D_BEFORE Y Xa)) (D_BEFORE Xa Z)) t ∪
          DFT_event p (D_AND Xa (D_BEFORE Y Xa)) t ∩
          (p_space p DIFF DFT_event p Z t) ∪
          DFT_event p
            (D_AND (D_AND Z (D_BEFORE Xd Y)) (D_BEFORE Y Z)) t ∪
          DFT_event p (D_AND Y (D_BEFORE Xd Y)) t ∩
          (p_space p DIFF DFT_event p Z t) ∪
          DFT_event p (D_AND Y (D_BEFORE Z Y)) t)
```

[shared_spare_disjoint_2]

```
 ⊢ ∀Xa Xd Y Z p t.
       rv_gt0_ninfinity [Xa; Xd; Y; Z] ∧
       (∀s. ALL_DISTINCT [Y s; Xa s; Xd s; Z s]) ∧
       DISJOINT_WSP Y Xa Xd t ∧ DISJOINT_WSP Z Xa Xd t ∧
       (∀s. D_AND (D_BEFORE Z Xd) (D_BEFORE Xd Y) s = NEVER s) ∧
```

```
(∀s. D_AND (D_BEFORE Y Xd) (D_BEFORE Xd Z) s = NEVER s) ∧
(∀s. D_AND (D_BEFORE Xa Y) (D_BEFORE Xa Z) s = NEVER s) ⇒
DISJOINT
  (DFT_event p
     (D_AND (D_AND Z (D_BEFORE Y Xa)) (D_BEFORE Xa Z)) t)
  (DFT_event p (D_AND Xa (D_BEFORE Y Xa)) t ∩
   (p_space p DIFF DFT_event p Z t) ∪
   DFT_event p
     (D_AND (D_AND Z (D_BEFORE Xd Y)) (D_BEFORE Y Z)) t ∪
   DFT_event p (D_AND Y (D_BEFORE Xd Y)) t ∩
   (p_space p DIFF DFT_event p Z t) ∪
   DFT_event p (D_AND Y (D_BEFORE Z Y)) t)
```

[shared_spare_disjoint_3]

```
⊢ ∀Xa Xd Y Z p t.
    rv_gt0_ninfinity [Xa; Xd; Y; Z] ∧
    (∀s. ALL_DISTINCT [Y s; Xa s; Xd s; Z s]) ∧
    DISJOINT_WSP Y Xa Xd t ∧ DISJOINT_WSP Z Xa Xd t ∧
    (∀s. D_AND (D_BEFORE Z Xd) (D_BEFORE Xd Y) s = NEVER s) ∧
    (∀s. D_AND (D_BEFORE Y Xd) (D_BEFORE Xd Z) s = NEVER s) ∧
    (∀s. D_AND (D_BEFORE Xa Y) (D_BEFORE Xa Z) s = NEVER s) ⇒
    DISJOINT
      (DFT_event p (D_AND Xa (D_BEFORE Y Xa)) t ∩
       (p_space p DIFF DFT_event p Z t))
      (DFT_event p
         (D_AND (D_AND Z (D_BEFORE Xd Y)) (D_BEFORE Y Z)) t ∪
       DFT_event p (D_AND Y (D_BEFORE Xd Y)) t ∩
       (p_space p DIFF DFT_event p Z t) ∪
       DFT_event p (D_AND Y (D_BEFORE Z Y)) t)
```

[shared_spare_disjoint_4]

```
⊢ ∀Xa Xd Y Z p t.
    DISJOINT
      (DFT_event p
         (D_AND (D_AND Z (D_BEFORE Xd Y)) (D_BEFORE Y Z)) t)
      (DFT_event p (D_AND Y (D_BEFORE Xd Y)) t ∩
       (p_space p DIFF DFT_event p Z t) ∪
       DFT_event p (D_AND Y (D_BEFORE Z Y)) t)
```

[shared_spare_disjoint_5]

```
⊢ ∀Xa Xd Y Z p t.
    DISJOINT
      (DFT_event p (D_AND Y (D_BEFORE Xd Y)) t ∩
       (p_space p DIFF DFT_event p Z t))
      (DFT_event p (D_AND Y (D_BEFORE Z Y)) t)
```

[shared_spare_Q1_event_distinct]

```
⊢ ∀ Xa Xd Y Z p t.
      rv_gt0_ninfinity [Xa; Xd; Y; Z] ∧
      (∀ s. ALL_DISTINCT [Y s; Xa s; Xd s; Z s]) ∧
      DISJOINT_WSP Y Xa Xd t ∧ DISJOINT_WSP Z Xa Xd t ∧
      (∀ s. D_AND (D_BEFORE Z Xd) (D_BEFORE Xd Y) s = NEVER s) ∧
      (∀ s. D_AND (D_BEFORE Y Xd) (D_BEFORE Xd Z) s = NEVER s) ∧
      (∀ s. D_AND (D_BEFORE Xa Y) (D_BEFORE Xa Z) s = NEVER s) ⇒
      (DFT_event p
         (D_OR
            (D_OR (D_AND Xa (D_BEFORE Y Xa))
               (D_AND Y (D_BEFORE Xd Y)))
            (D_AND Y (D_BEFORE Z Y))) t =
       DFT_event p
         (D_AND (D_AND Xa (D_BEFORE Y Z)) (D_BEFORE Z Xa)) t ∪
       DFT_event p
         (D_AND (D_AND Z (D_BEFORE Y Xa)) (D_BEFORE Xa Z)) t ∪
       DFT_event p (D_AND Xa (D_BEFORE Y Xa)) t ∩
       (p_space p DIFF DFT_event p Z t) ∪
       DFT_event p
         (D_AND (D_AND Z (D_BEFORE Xd Y)) (D_BEFORE Y Z)) t ∪
       DFT_event p (D_AND Y (D_BEFORE Xd Y)) t ∩
       (p_space p DIFF DFT_event p Z t) ∪
       DFT_event p (D_AND Y (D_BEFORE Z Y)) t)
```