1 DRBD Theory

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Parent Theories: WCSP

1.1 Definitions

[BIGUNION_o_BIGUNION_def]

$\vdash \forall s\ L\ A.$
$\quad \texttt{BIGUNION\_o\_BIGUNION}\ s\ L\ A =$
$\quad \texttt{BIGUNION}\ \{s\ l \mid l \in \texttt{BIGUNION}\ \{L\ a \mid a \in A\}\}$

[DRBD_event_def]

$\vdash \forall p\ X\ t.\ \texttt{DRBD\_event}\ p\ X\ t = \{s \mid \texttt{Normal}\ t < X\ s\} \cap \texttt{p\_space}\ p$

[DRBD_parallel_def]

$\vdash \forall Y\ s.\ \texttt{DRBD\_parallel}\ Y\ s = \texttt{BIGUNION}\ \{Y\ i \mid i \in s\}$

[DRBD_series_def]

$\vdash \forall Y\ s.\ \texttt{DRBD\_series}\ Y\ s = \texttt{BIGINTER}\ \{Y\ i \mid i \in s\}$

[from_nat_into_def]

$\vdash \forall A.$
$\quad \texttt{from\_nat\_into}\ A =$
$\quad (\lambda n.$
$\quad\quad \textbf{if}\ n \in \texttt{IMAGE}\ (\texttt{to\_nat\_on}\ A)\ A\ \textbf{then}$
$\quad\quad\quad \texttt{inv\_into}\ A\ (\texttt{to\_nat\_on}\ A)\ n$
$\quad\quad \textbf{else}\ \varepsilon s.\ s \in A)$

[inv_into_def]

$\vdash \forall s\ f.\ \texttt{inv\_into}\ s\ f = (\lambda y.\ \texttt{LINV}\ f\ s\ y)$

[nested_BIGUNION_def]

$\vdash \forall s\ L\ A\ J.$
$\quad \texttt{nested\_BIGUNION}\ s\ L\ A\ J =$
$\quad \texttt{BIGUNION}$
$\quad\quad \{s\ l \mid l \in \texttt{BIGUNION}\ \{L\ a \mid a \in \texttt{BIGUNION}\ \{A\ j \mid j \in J\}\}\}$

[nR_AND_def]

$\vdash \forall X\ s.\ \texttt{nR\_AND}\ X\ s = \texttt{ITSET}\ (\lambda e\ acc.\ \texttt{R\_AND}\ (X\ e)\ acc)\ s\ \texttt{R\_NEVER}$

[nR_OR_def]

$\vdash \forall X\ s.\ \texttt{nR\_OR}\ X\ s = \texttt{ITSET}\ (\lambda e\ acc.\ \texttt{R\_OR}\ (X\ e)\ acc)\ s\ \texttt{R\_ALWAYS}$

[R_AFTER_def]

$\vdash \forall X\ Y.\ \texttt{R\_AFTER}\ X\ Y = (\lambda s.\ \textbf{if}\ Y\ s < X\ s\ \textbf{then}\ X\ s\ \textbf{else}\ \texttt{PosInf})$

[R_ALWAYS_def]

$\vdash$ R_ALWAYS = (λs. 0)

[R_AND_def]

$\vdash$ $\forall X$ Y. R_AND X Y = (λs. min (X s) (Y s))

[R_BEFORE_def]

$\vdash$ $\forall X$ Y. R_BEFORE X Y = (λs. **if** X s $<$ Y s **then** X s **else** PosInf)

[R_CSP_def]

$\vdash$ $\forall Y$ X. R_CSP Y X = (λs. **if** Y s $<$ X s **then** X s **else** PosInf)

[R_HSP_def]

$\vdash$ $\forall Y$ X. R_HSP Y X = (λs. max (Y s) (X s))

[R_INCLUSIVE_AFTER_def]

$\vdash$ $\forall X$ Y.
R_INCLUSIVE_AFTER X Y =
(λs. **if** Y s $\leq$ X s **then** X s **else** PosInf)

[R_INCLUSIVE_BEFORE_def]

$\vdash$ $\forall X$ Y.
R_INCLUSIVE_BEFORE X Y =
(λs. **if** X s $\leq$ Y s **then** X s **else** PosInf)

[R_NEVER_def]

$\vdash$ R_NEVER = (λs. PosInf)

[R_OR_def]

$\vdash$ $\forall X$ Y. R_OR X Y = (λs. max (X s) (Y s))

[R_SIMULT_def]

$\vdash$ $\forall X$ Y. R_SIMULT X Y = (λs. **if** X s = Y s **then** X s **else** PosInf)

[R_WSP_def]

$\vdash$ $\forall Y$ X_a X_d.
R_WSP Y X_a X_d = R_AND (R_AFTER X_a Y) (R_AFTER Y X_d)

[range_def]

$\vdash$ $\forall f$. range f = IMAGE f $\mathcal{U}$(:'a)

[Rel_def]

$\vdash$ $\forall p$ X t. Rel p X t = prob p (DRBD_event p X t)

[rv_to_event_def]

$\vdash \forall p\ X\ t.$ rv_to_event $p\ X\ t$ = $(\lambda i.$ DRBD_event $p\ (X\ i)\ t)$

[sets_finite_not_empty2_def]

$\vdash \forall L\ A\ J.$
sets_finite_not_empty2 $L\ A\ J \iff$
$(\forall i.\ i \in J \Rightarrow A\ i \neq \{\} \wedge$ FINITE $(A\ i)) \wedge$
$(\forall i.\ i \in$ BIGUNION $\{A\ j \mid j \in J\} \Rightarrow L\ i \neq \{\} \wedge$ FINITE $(L\ i)) \wedge$
FINITE $J \wedge$ disjoint_family_on $A\ J \wedge J \neq \{\} \wedge$
disjoint_family_on L (BIGUNION $\{A\ j \mid j \in J\}$)

[sets_finite_not_empty_def]

$\vdash \forall s\ L\ A\ J.$
sets_finite_not_empty $s\ L\ A\ J \iff$
$(\forall i.\ i \in J \Rightarrow A\ i \neq \{\} \wedge$ FINITE $(A\ i)) \wedge$
$(\forall i.$
$i \in$ BIGUNION $\{L\ a \mid a \in$ BIGUNION $\{A\ j \mid j \in J\}\} \Rightarrow$
$s\ i \neq \{\} \wedge$ FINITE $(s\ i)) \wedge$
$(\forall i.\ i \in$ BIGUNION $\{A\ j \mid j \in J\} \Rightarrow L\ i \neq \{\} \wedge$ FINITE $(L\ i)) \wedge$
FINITE $J \wedge$ disjoint_family_on $A\ J \wedge J \neq \{\} \wedge$
disjoint_family_on s
(BIGUNION $\{L\ a \mid a \in$ BIGUNION $\{A\ j \mid j \in J\}\}) \wedge$
disjoint_family_on L (BIGUNION $\{A\ j \mid j \in J\}$)

[to_nat_on_def]

$\vdash \forall A.$
to_nat_on A =
$\varepsilon f.$
if FINITE A **then** BIJ $f\ A$ (count (CARD A))
else BIJ $f\ A\ \mathcal{U}$(:num)

[UNIONL_def]

$\vdash$ (UNIONL [] = $\{\}$) $\wedge \forall s\ ss.$ UNIONL $(s::ss)$ = $s \cup$ UNIONL ss

1.2 Theorems

[BIGINTER_SUBSET_BIGUNION]

$\vdash \forall P.\ P \neq \{\} \Rightarrow$ BIGINTER $P \subseteq$ BIGUNION P

[BIGUNION_IN_events_p]

$\vdash \forall p\ X\ s.$
FINITE $s \wedge (\forall i.\ i \in s \Rightarrow X\ i \in$ events $p) \wedge$ prob_space $p \Rightarrow$
BIGUNION $\{X\ i \mid i \in s\} \in$ events p

[BIJ_CONG]

$\vdash \forall s\ t\ f\ g.$
$(\forall a.\ a \in s \Rightarrow (f\ a = g\ a)) \Rightarrow$ (BIJ $f\ s\ t \iff$ BIJ $g\ s\ t$)

[CARD_EQ_BIJ]

⊢ $\forall s\ t.$ FINITE s ∧ FINITE t ∧ (CARD s = CARD t) ⇒ $\exists f.$ BIJ $f\ s\ t$

[CDF_def3]

⊢ $\forall p\ X\ x.$
cont_CDF $p\ X$ ∧ random_variable $X\ p$ borel ⇒
(CDF $p\ X\ x$ = prob p (PREIMAGE X $\{y \mid y < x\}$ ∩ p_space p))

[CDF_Rel]

⊢ $\forall p\ X\ t.$
rv_gt0_ninfinity [X] ∧ random_variable (real ∘ X) p borel ⇒
(Rel $p\ X\ t$ = 1 - CDF p (real ∘ X) t)

[countableE_infinite]

⊢ $\forall A.$ countable A ∧ INFINITE A ⇒ $\exists f.$ BIJ $f\ A\ \mathcal{U}$(:num)

[DFT_event_in_event_p]

⊢ $\forall X\ p\ t.$
rv_gt0_ninfinity [X] ∧
random_variable ($\lambda s.$ real ($X\ s$)) p borel ⇒
DFT_event $p\ X\ t$ ∈ events p

[DIFF_BIGINTER2]

⊢ $\forall sp\ s.$
($\forall t.\ t \in s \Rightarrow t \subseteq sp$) ∧ $s \neq \{\}$ ⇒
(BIGINTER s = sp DIFF BIGUNION (IMAGE ($\lambda u.$ sp DIFF u) s))

[DIFF_BIGUNION_BIGINTER]

⊢ $\forall p\ X\ s.$
$s \neq \{\}$ ∧ ($\forall i.\ i \in s \Rightarrow X\ i \in$ events p) ∧ prob_space p ⇒
(p_space p DIFF BIGUNION $\{X\ i \mid i \in s\}$ =
BIGINTER {p_space p DIFF $X\ i \mid i \in s$})

[DRBD_event_in_event_p]

⊢ $\forall X\ p\ t.$
rv_gt0_ninfinity [X] ∧
random_variable ($\lambda s.$ real ($X\ s$)) p borel ⇒
DRBD_event $p\ X\ t$ ∈ events p

[DRBD_event_INTER_p_space_p]

⊢ $\forall p\ X\ t.$ DRBD_event $p\ X\ t$ ∩ p_space p = DRBD_event $p\ X\ t$

[DRBD_event_PREIMAGE]

⊢ $\forall X\ p\ t.$
DRBD_event $p\ X\ t$ =
PREIMAGE X {s | Normal $t < s$} ∩ p_space p

[DRBD_parallel_IN_events_p]
```
⊢ ∀p X s.
      prob_space p ∧ FINITE s ∧ (∀i. i ∈ s ⇒ X i ∈ events p) ⇒
      DRBD_parallel X s ∈ events p
```

[DRBD_series_AND_INRER_3]
```
⊢ ∀X Y Z p t.
      DRBD_event p (R_AND (R_AND X Y) Z) t =
      DRBD_series
        (λi.
             if i = 0 then DRBD_event p X t
             else if i = 1 then DRBD_event p Y t
             else DRBD_event p Z t) {0; 1; 2}
```

[DRBD_series_AND_INTER]
```
⊢ ∀X Y p t.
      DRBD_event p (R_AND X Y) t =
      DRBD_series
        (λi.
             if i = 0 then DRBD_event p X t
             else DRBD_event p Y t) {0; 1}
```

[DRBD_series_IN_events_p]
```
⊢ ∀p X s.
      prob_space p ∧ (∀i. i ∈ s ⇒ X i ∈ events p) ∧
      countable s ∧ s ≠ {} ⇒
      DRBD_series X s ∈ events p
```

[DRBD_series_parallel_series_lem]
```
⊢ ∀p X L A J.
      DRBD_series
        (λj.
             DRBD_parallel
               (λa. DRBD_series (λl. DRBD_parallel X {l}) (L a))
               (A j)) J =
      DRBD_series
        (λj. DRBD_parallel (λa. DRBD_series X (L a)) (A j)) J
```

[DRBD_series_parallel_series_lem2]
```
⊢ ∀p X s L A J.
      (∀l. l ∈ BIGUNION_o_BIGUNION L A J ⇒ (s l = {l})) ⇒
      (DRBD_series
         (λj.
              DRBD_parallel
                (λa.
                     DRBD_series (λl. DRBD_parallel X (s l))
                       (L a)) (A j)) J =
       DRBD_series
         (λj. DRBD_parallel (λa. DRBD_series X (L a)) (A j)) J)
```

[extreal_sub_sub2]

$\vdash \forall a\ b.$
$a \neq$ PosInf $\wedge\ a \neq$ NegInf $\wedge\ b \neq$ PosInf $\wedge\ b \neq$ NegInf $\Rightarrow$
$(a$ - $(a$ - $b)$ = $b)$

[FDEP_R_AND]

$\vdash \forall X\ Y\ p\ t.$
DFT_event p (FDEP X Y) t =
p_space p DIFF DRBD_event p (R_AND X Y) t

[finite_imageD]

$\vdash \forall f\ A.$ FINITE (IMAGE f A) $\wedge$ INJ f A (IMAGE f A) $\Rightarrow$ FINITE A

[finite_imageD2]

$\vdash \forall f\ A\ t.$ FINITE (IMAGE f A) $\wedge$ INJ f A t $\Rightarrow$ FINITE A

[finite_imageD_lem]

$\vdash \forall f\ A\ B.$
FINITE B $\Rightarrow$
(B = IMAGE f A) $\wedge$ INJ f A (IMAGE f A) $\Rightarrow$
FINITE A

[finite_imageD_lem2]

$\vdash \forall f\ A\ t\ B.$ FINITE B $\Rightarrow$ (B = IMAGE f A) $\wedge$ INJ f A t $\Rightarrow$ FINITE A

[from_nat_into_lem]

$\vdash \forall A\ n.\ A \neq \{\,\} \Rightarrow$ from_nat_into A n $\in A$

[from_nat_into_nat_on]

$\vdash \forall A\ a.$
countable A $\wedge\ a \in A \Rightarrow$
(from_nat_into A (to_nat_on A a) = a)

[HSP_OR]

$\vdash \forall X\ Y.$ R_HSP Y X = R_OR Y X

[IN_REST]

$\vdash \forall x\ s.\ x \in$ REST s $\iff$ $x \in s \wedge x \neq$ CHOICE s

[IN_UNIONL]

$\vdash \forall l\ v.\ v \in$ UNIONL l $\iff$ $\exists s.$ MEM s l $\wedge\ v \in s$

[indep_sets_BIGUNION_BIGINTER]

$\vdash \forall E\ J\ s\ p.$
$(\forall i.\ s\ i \neq \{\,\})$ $\wedge$
indep_sets p ($\lambda j.$ BIGUNION $\{E\ i \mid i \in s\ j\}$) J $\Rightarrow$
indep_sets p ($\lambda j.$ BIGINTER $\{E\ i \mid i \in s\ j\}$) J

[indep_sets_collect_sigma_BIGINTER_events]

$\vdash \forall p\ s\ J\ Y.$
    indep_sets p ($\lambda i.$ $\{Y\ i\}$) (BIGUNION $\{s\ j \mid j \in J\}$) $\wedge$
    $J \neq \{\}$ $\wedge$ ($\forall i.$ $i \in J \Rightarrow$ countable ($s\ i$) $\wedge$ $s\ i \neq \{\}$) $\wedge$
    FINITE J $\wedge$ disjoint_family_on $s\ J$ $\wedge$
    ($\forall i.$ $i \in$ BIGUNION $\{s\ j \mid j \in J\} \Rightarrow Y\ i \subseteq$ m_space p) $\Rightarrow$
    indep_sets p ($\lambda j.$ $\{$BIGINTER $\{Y\ i \mid i \in s\ j\}\}$) J

[indep_sets_collect_sigma_BIGUNION]

$\vdash \forall p\ E\ J\ s.$
    indep_sets p
      ($\lambda j.$ sigma_sets (m_space p) (BIGUNION $\{E\ i \mid i \in s\ j\}$))
      $J \Rightarrow$
    indep_sets p ($\lambda j.$ BIGUNION $\{E\ i \mid i \in s\ j\}$) J

[indep_sets_collect_sigma_BIGUNION_events]

$\vdash \forall p\ s\ J\ Y.$
    indep_sets p ($\lambda i.$ $\{Y\ i\}$) (BIGUNION $\{s\ j \mid j \in J\}$) $\wedge$
    $J \neq \{\}$ $\wedge$ ($\forall i.$ $i \in J \Rightarrow$ countable ($s\ i$)) $\wedge$ FINITE J $\wedge$
    disjoint_family_on $s\ J \Rightarrow$
    indep_sets p ($\lambda j.$ $\{$BIGUNION $\{Y\ i \mid i \in s\ j\}\}$) J

[indep_sets_sigma_1]

$\vdash \forall p\ X\ s.$
    indep_sets p ($\lambda i.$ $\{X\ i\}$) $s \Rightarrow$
    indep_sets p ($\lambda i.$ sigma_sets (p_space p) $\{X\ i\}$) s

[indep_sets_sigma_compl]

$\vdash \forall p\ E\ J.$
    indep_sets p ($\lambda j.$ sigma_sets (m_space p) $\{E\ j\}$) $J \Rightarrow$
    indep_sets p ($\lambda j.$ $\{$m_space p DIFF $E\ j\}$) J

[INT_from_nat_into]

$\vdash \forall s\ A.$
    $s \neq \{\}$ $\wedge$ countable $s \Rightarrow$
    (BIGINTER $\{A\ i \mid i \in s\}$ =
     BIGINTER $\{A$ (from_nat_into $s\ i$) $\mid i \in \mathcal{U}$(:num)$\}$)

[inv_into_f_f]

$\vdash \forall A\ f\ x\ t.$ INJ $f\ A\ t \wedge x \in A \Rightarrow$ (inv_into $A\ f$ ($f\ x$) = x)

[inv_into_into]

$\vdash \forall x\ f\ A.$ $x \in$ IMAGE $f\ A \Rightarrow$ inv_into $A\ f\ x \in A$

[lborel_ge]

$\vdash \forall x.$ $\{w \mid x \leq w\} \in$ measurable_sets lborel

[lborel_gt]

$\vdash \forall x.\ \{w \mid x < w\} \in$ measurable_sets lborel

[normal_real_mul1]

$\vdash \forall a\ b\ c.$
$a \neq$ PosInf $\wedge$ $a \neq$ NegInf $\wedge$ $b \neq$ PosInf $\wedge$ $b \neq$ NegInf $\Rightarrow$
(Normal (real $a \times$ real $b \times c$) = $a \times b \times$ Normal c)

[normal_real_mul2]

$\vdash \forall a\ b\ c\ d.$
$a \neq$ PosInf $\wedge$ $a \neq$ NegInf $\wedge$ $b \neq$ PosInf $\wedge$ $b \neq$ NegInf $\Rightarrow$
(Normal (real $a \times c \times d \times$ real b) =
$a \times b \times$ Normal ($c \times d$))

[normal_real_mul3]

$\vdash \forall a\ b\ c.$
$a \neq$ PosInf $\wedge$ $a \neq$ NegInf $\wedge$ $b \neq$ PosInf $\wedge$ $b \neq$ NegInf $\Rightarrow$
(Normal (real $a \times c \times$ real b) = $a \times b \times$ Normal c)

[normal_real_product]

```
⊢ ∀f J s.
      Normal
        (product J
           (λj.
                real
                  (1 -
                   Normal (product (s j) (λi. real (1 - f i)))))) =
      Normal
        (product J (λj. 1 - product (s j) (λi. real (1 - f i))))
```

[normal_real_product_1]

```
⊢ ∀f J s.
      1 -
      Normal
        (product J
           (λj.
                real
                  (1 - Normal (product (s j) (λi. real (f i)))))) =
      1 -
      Normal
        (product J (λj. 1 - product (s j) (λi. real (f i))))
```

[normal_real_product_2]

```
⊢ ∀f s.
      real (1 - Normal (product s (λi. f i))) =
      1 - product s (λi. f i)
```

[nR_AND_DRBD1]

$\vdash \forall p\ X\ t\ s.$
    FINITE $s \Rightarrow$
    (DRBD_event p (nR_AND X s) t =
     DRBD_series (rv_to_event p X t) $s \cap$ p_space p)

[nR_AND_DRBD_series]

$\vdash \forall p\ X\ t\ s.$
    FINITE $s \wedge s \neq \{\} \Rightarrow$
    (DRBD_event p (nR_AND X s) t =
     DRBD_series (rv_to_event p X t) s)

[nR_AND_THM]

$\vdash \forall X.$
    (nR_AND X $\{\}$ = ($\lambda s.$ PosInf)) $\wedge$
    $\forall e\ s.$
        FINITE $s \Rightarrow$
        (nR_AND X (e INSERT s) =
         R_AND (X e) (nR_AND X (s DELETE e)))

[nR_OR_DRBD1]

$\vdash \forall p\ X\ t\ s.$
    FINITE $s \Rightarrow$
    $0 \leq t \Rightarrow$
    (DRBD_event p (nR_OR X s) t =
     DRBD_parallel (rv_to_event p X t) s)

[nR_OR_DRBD_parallel]

$\vdash \forall p\ X\ t\ s.$
    FINITE $s \wedge 0 \leq t \Rightarrow$
    (DRBD_event p (nR_OR X s) t =
     DRBD_parallel (rv_to_event p X t) s)

[nR_OR_THM]

$\vdash \forall X.$
    (nR_OR X $\{\}$ = ($\lambda s.$ 0)) $\wedge$
    $\forall e\ s.$
        FINITE $s \Rightarrow$
        (nR_OR X (e INSERT s) =
         R_OR (X e) (nR_OR X (s DELETE e)))

[PAND_R_AFTER]

$\vdash \forall X\ Y\ p\ t.$
    ($\forall s.$ ALL_DISTINCT [X s; Y s]) $\Rightarrow$
    (DFT_event p (P_AND Y X) t =
     p_space p DIFF DRBD_event p (R_AFTER X Y) t)

[PREIMAGE_EXTREAL_REAL_le]

$\vdash \forall X\ t\ p.$
$\quad(\forall s.\ X\ s \neq \texttt{PosInf} \land 0 \leq X\ s) \Rightarrow$
$\quad(\texttt{PREIMAGE}\ X\ \{u \mid \texttt{Normal}\ t \leq u\} \cap \texttt{p\_space}\ p =$
$\quad\ \texttt{PREIMAGE}\ (\lambda s.\ \texttt{real}\ (X\ s))\ \{u \mid t \leq u\} \cap \texttt{p\_space}\ p)$

[PREIMAGE_EXTREAL_REAL_lt]

$\vdash \forall X\ t\ p.$
$\quad(\forall s.\ X\ s \neq \texttt{PosInf} \land 0 \leq X\ s) \Rightarrow$
$\quad(\texttt{PREIMAGE}\ X\ \{u \mid \texttt{Normal}\ t < u\} \cap \texttt{p\_space}\ p =$
$\quad\ \texttt{PREIMAGE}\ (\lambda s.\ \texttt{real}\ (X\ s))\ \{u \mid t < u\} \cap \texttt{p\_space}\ p)$

[PREIMAGE_EXTREAL_REAL_lt1]

$\vdash \forall X\ t\ p.$
$\quad(\forall s.\ X\ s \neq \texttt{PosInf} \land 0 \leq X\ s) \Rightarrow$
$\quad(\texttt{PREIMAGE}\ X\ \{u \mid u < \texttt{Normal}\ t\} \cap \texttt{p\_space}\ p =$
$\quad\ \texttt{PREIMAGE}\ (\lambda s.\ \texttt{real}\ (X\ s))\ \{u \mid u < t\} \cap \texttt{p\_space}\ p)$

[PROB_BIGINTER_COMPL]

$\vdash \forall p\ X\ s.$
$\quad\texttt{indep\_sets}\ p\ (\lambda i.\ \{X\ i\})\ s \land \texttt{FINITE}\ s \land s \neq \{\,\} \land$
$\quad(\forall i.\ i \in s \Rightarrow X\ i \in \texttt{events}\ p) \Rightarrow$
$\quad(\texttt{prob}\ p\ (\texttt{BIGINTER}\ \{\texttt{p\_space}\ p\ \texttt{DIFF}\ X\ i \mid i \in s\}) =$
$\quad\ \texttt{Normal}\ (\texttt{product}\ s\ (\lambda i.\ \texttt{real}\ (1 - \texttt{prob}\ p\ (X\ i)))))$

[PROB_DFT_DRBD_COMP]

$\vdash \forall X\ p\ t.$
$\quad\texttt{rv\_gt0\_ninfinity}\ [X] \land$
$\quad\texttt{random\_variable}\ (\lambda s.\ \texttt{real}\ (X\ s))\ p\ \texttt{borel} \Rightarrow$
$\quad(\texttt{prob}\ p\ (\texttt{DFT\_event}\ p\ X\ t) = 1 - \texttt{prob}\ p\ (\texttt{DRBD\_event}\ p\ X\ t))$

[PROB_DRBD_DFT_COMP]

$\vdash \forall X\ p\ t.$
$\quad\texttt{rv\_gt0\_ninfinity}\ [X] \land$
$\quad\texttt{random\_variable}\ (\lambda s.\ \texttt{real}\ (X\ s))\ p\ \texttt{borel} \Rightarrow$
$\quad(\texttt{prob}\ p\ (\texttt{DRBD\_event}\ p\ X\ t) = 1 - \texttt{prob}\ p\ (\texttt{DFT\_event}\ p\ X\ t))$

[PROB_DRBD_nested_series_parallel]

$\vdash \forall p\ X\ s\ L\ A\ J.$
$\quad(\forall i.\ i \in \texttt{nested\_BIGUNION}\ s\ L\ A\ J \Rightarrow X\ i \in \texttt{events}\ p) \land$
$\quad\texttt{indep\_sets}\ p\ (\lambda i.\ \{X\ i\})\ (\texttt{nested\_BIGUNION}\ s\ L\ A\ J) \land$
$\quad\texttt{sets\_finite\_not\_empty}\ s\ L\ A\ J \Rightarrow$
$\quad(\texttt{prob}\ p$
$\quad\quad(\texttt{DRBD\_series}$
$\quad\quad\quad(\lambda j.$
$\quad\quad\quad\quad\texttt{DRBD\_parallel}$
$\quad\quad\quad\quad\ (\lambda a.$

```
                        DRBD_series (λl. DRBD_parallel X (s l))
                          (L a)) (A j)) J) =
        Normal
          (product J
             (λj.
                  real
                    (1 -
                     Normal
                       (product (A j)
                          (λa.
                               real
                                 (1 -
                                  Normal
                                    (product (L a)
                                       (λl.
                                            real
                                              (1 -
                                               Normal
                                                 (product (s l)
                                                    (λi.
                                                         real
                                                           (1 -
                                                            prob
                                                              p
                                                              (X
                                                                 i))))))))))))))
```

[PROB_DRBD_nested_series_parallel_red]

```
 ⊢ ∀p X s L A J.
       (∀i. i ∈ nested_BIGUNION s L A J ⇒ X i ∈ events p) ∧
       indep_sets p (λi. {X i}) (nested_BIGUNION s L A J) ∧
       sets_finite_not_empty s L A J ⇒
       (prob p
          (DRBD_series
             (λj.
                  DRBD_parallel
                    (λa.
                         DRBD_series (λl. DRBD_parallel X (s l))
                           (L a)) (A j)) J) =
        Normal
          (product J
             (λj.
                  1 -
                  product (A j)
                    (λa.
                         1 -
                         product (L a)
                           (λl.
                                1 -
```

```
                                        product (s l)
                                          (λ i. real (1 - prob p (X i)))))))
```

[PROB_DRBD_nested_series_parallel_rv]

```
 ⊢ ∀ p X t s L A J.
        (∀ i.
             i ∈ nested_BIGUNION s L A J ⇒
             rv_to_event p X t i ∈ events p) ∧
        indep_sets p (λ i. {rv_to_event p X t i})
          (nested_BIGUNION s L A J) ∧
        sets_finite_not_empty s L A J ⇒
        (prob p
           (DRBD_series
              (λ j.
                     DRBD_parallel
                       (λ a.
                              DRBD_series
                                (λ l.
                                       DRBD_parallel
                                         (rv_to_event p X t) (s l))
                                (L a)) (A j)) J) =
         Normal
           (product J
              (λ j.
                     real
                       (1 -
                        Normal
                          (product (A j)
                             (λ a.
                                    real
                                      (1 -
                                       Normal
                                         (product (L a)
                                            (λ l.
                                                   real
                                                     (1 -
                                                      Normal
                                                        (product (s l)
                                                           (λ i.
                                                                  real
                                                                    (1 -
                                                                     Rel
                                                                       p
                                                                       (X
                                                                          i)
                                                                       t))))))))))))
```

[PROB_DRBD_nested_series_parallel_rv_red]

```
 ⊢ ∀ p X t s L A J.
        (∀ i.
```

```
         i ∈ nested_BIGUNION s L A J ⇒
         rv_to_event p X t i ∈ events p) ∧
    indep_sets p (λi. {rv_to_event p X t i})
      (nested_BIGUNION s L A J) ∧
    sets_finite_not_empty s L A J ⇒
    (prob p
       (DRBD_series
          (λj.
               DRBD_parallel
                 (λa.
                      DRBD_series
                        (λl.
                             DRBD_parallel
                               (rv_to_event p X t) (s l))
                        (L a)) (A j)) J) =
     Normal
       (product J
          (λj.
               1 -
               product (A j)
                 (λa.
                      1 -
                      product (L a)
                        (λl.
                             1 -
                             product (s l)
                               (λi. real (1 - Rel p (X i) t)))))))
```

[PROB_DRBD_parallel]

```
 ⊢ ∀p X s.
     indep_sets p (λi. {X i}) s ∧ s ≠ {} ∧ FINITE s ∧
     (∀i. i ∈ s ⇒ X i ∈ events p) ⇒
     (prob p (DRBD_parallel X s) =
      1 - Normal (product s (λi. real (1 - prob p (X i)))))
```

[PROB_DRBD_parallel_BIGINTER]

```
 ⊢ ∀p X s.
     indep_sets p (λi. {X i}) s ∧ FINITE s ∧ s ≠ {} ∧
     (∀i. i ∈ s ⇒ X i ∈ events p) ⇒
     (prob p (DRBD_parallel X s) =
      1 - prob p (BIGINTER {p_space p DIFF X i | i ∈ s}))
```

[PROB_DRBD_parallel_compl]

```
 ⊢ ∀p X s.
     indep_sets p (λi. {X i}) s ∧ FINITE s ∧
     (∀i. i ∈ s ⇒ X i ∈ events p) ⇒
     (prob p (DRBD_parallel X s) =
      1 - prob p (p_space p DIFF BIGUNION {X i | i ∈ s}))
```

[PROB_DRBD_parallel_rv]

```
⊢ ∀p X t s.
      s ≠ { } ∧ FINITE s ∧
      indep_sets p (λi. {rv_to_event p X t i}) s ∧
      (∀i. i ∈ s ⇒ rv_to_event p X t i ∈ events p) ⇒
      (prob p (DRBD_parallel (rv_to_event p X t) s) =
       1 - Normal (product s (λi. real (1 - Rel p (X i) t))))
```

[PROB_DRBD_parallel_series]

```
⊢ ∀p X s J.
      indep_sets p (λi. {X i}) (BIGUNION {s j | j ∈ J}) ∧
      (∀i. i ∈ J ⇒ countable (s i) ∧ s i ≠ { } ∧ FINITE (s i)) ∧
      FINITE J ∧
      (∀i. i ∈ BIGUNION {s j | j ∈ J} ⇒ X i ∈ events p) ∧
      disjoint_family_on s J ∧ J ≠ { } ⇒
      (prob p (DRBD_parallel (λj. DRBD_series X (s j)) J) =
       1 -
       Normal
         (product J
            (λj.
                 real
                   (1 -
                    Normal
                      (product (s j) (λi. real (prob p (X i))))))))
```

[PROB_DRBD_parallel_series_red]

```
⊢ ∀p X s J.
      indep_sets p (λi. {X i}) (BIGUNION {s j | j ∈ J}) ∧
      (∀i. i ∈ J ⇒ countable (s i) ∧ s i ≠ { } ∧ FINITE (s i)) ∧
      FINITE J ∧
      (∀i. i ∈ BIGUNION {s j | j ∈ J} ⇒ X i ∈ events p) ∧
      disjoint_family_on s J ∧ J ≠ { } ⇒
      (prob p (DRBD_parallel (λj. DRBD_series X (s j)) J) =
       1 -
       Normal
         (product J
            (λj. 1 - product (s j) (λi. real (prob p (X i))))))
```

[PROB_DRBD_parallel_series_rv]

```
⊢ ∀p X t s J.
      indep_sets p (λi. {rv_to_event p X t i})
        (BIGUNION {s j | j ∈ J}) ∧
      (∀i. i ∈ J ⇒ countable (s i) ∧ s i ≠ { } ∧ FINITE (s i)) ∧
      FINITE J ∧
      (∀i.
           i ∈ BIGUNION {s j | j ∈ J} ⇒
           rv_to_event p X t i ∈ events p) ∧
      disjoint_family_on s J ∧ J ≠ { } ⇒
```

```
        (prob p
           (DRBD_parallel
              (λj. DRBD_series (rv_to_event p X t) (s j)) J) =
         1 -
         Normal
           (product J
              (λj.
                   real
                     (1 -
                      Normal
                        (product (s j) (λi. real (Rel p (X i) t)))))))
```

[PROB_DRBD_parallel_series_rv_red]

```
 ⊢ ∀p X t s J.
        indep_sets p (λi. {rv_to_event p X t i})
          (BIGUNION {s j | j ∈ J}) ∧
        (∀i.
             i ∈ BIGUNION {s j | j ∈ J} ⇒
             rv_to_event p X t i ∈ events p) ∧
        (∀i. i ∈ J ⇒ s i ≠ {} ∧ FINITE (s i)) ∧ FINITE J ∧
        J ≠ {} ∧ disjoint_family_on s J ⇒
        (prob p
           (DRBD_parallel
              (λj. DRBD_series (rv_to_event p X t) (s j)) J) =
         1 -
         Normal
           (product J
              (λj. 1 - product (s j) (λi. real (Rel p (X i) t)))))
```

[PROB_DRBD_series]

```
 ⊢ ∀p X s.
        indep_sets p (λi. {X i}) s ∧ s ≠ {} ∧ FINITE s ⇒
        (prob p (DRBD_series X s) =
         Normal (product s (λi. real (prob p (X i)))))
```

[PROB_DRBD_series_parallel]

```
 ⊢ ∀p X s J.
        indep_sets p (λi. {X i}) (BIGUNION {s j | j ∈ J}) ∧
        (∀i. i ∈ J ⇒ countable (s i) ∧ s i ≠ {} ∧ FINITE (s i)) ∧
        FINITE J ∧ disjoint_family_on s J ∧ J ≠ {} ⇒
        (prob p (DRBD_series (λj. DRBD_parallel X (s j)) J) =
         Normal
           (product J
              (λj.
                   real
                     (1 -
                      Normal
                        (product (s j)
                           (λi. real (1 - prob p (X i))))))))
```

[PROB_DRBD_series_parallel_red]

```
⊢ ∀p X s J.
      indep_sets p (λi. {X i}) (BIGUNION {s j | j ∈ J}) ∧
      (∀i. i ∈ J ⇒ countable (s i) ∧ s i ≠ {} ∧ FINITE (s i)) ∧
      FINITE J ∧ disjoint_family_on s J ∧ J ≠ {} ⇒
      (prob p (DRBD_series (λj. DRBD_parallel X (s j)) J) =
       Normal
         (product J
            (λj.
                 1 -
                 product (s j) (λi. real (1 - prob p (X i))))))
```

[PROB_DRBD_series_parallel_rv]

```
⊢ ∀p X t s J.
      indep_sets p (λi. {rv_to_event p X t i})
        (BIGUNION {s j | j ∈ J}) ∧
      (∀i. i ∈ J ⇒ countable (s i) ∧ s i ≠ {} ∧ FINITE (s i)) ∧
      FINITE J ∧ disjoint_family_on s J ∧ J ≠ {} ⇒
      (prob p
         (DRBD_series
            (λj. DRBD_parallel (rv_to_event p X t) (s j)) J) =
       Normal
         (product J
            (λj.
                 real
                   (1 -
                    Normal
                      (product (s j)
                         (λi. real (1 - Rel p (X i) t)))))))
```

[PROB_DRBD_series_parallel_rv_red]

```
⊢ ∀p X t s J.
      indep_sets p (λi. {rv_to_event p X t i})
        (BIGUNION {s j | j ∈ J}) ∧
      (∀i. i ∈ J ⇒ s i ≠ {} ∧ FINITE (s i)) ∧ FINITE J ∧
      J ≠ {} ∧ disjoint_family_on s J ⇒
      (prob p
         (DRBD_series
            (λj. DRBD_parallel (rv_to_event p X t) (s j)) J) =
       Normal
         (product J
            (λj.
                 1 -
                 product (s j) (λi. real (1 - Rel p (X i) t)))))
```

[PROB_DRBD_series_parallel_series]

```
⊢ ∀p X s L A J.
      (∀l. l ∈ BIGUNION_o_BIGUNION L A J ⇒ X l ∈ events p) ∧
```

```
(∀ l. l ∈ BIGUNION_o_BIGUNION L A J ⇒ (s l = {l})) ∧
indep_sets p (λ i. {X i}) (BIGUNION_o_BIGUNION L A J) ∧
sets_finite_not_empty s L A J ⇒
(prob p
   (DRBD_series
      (λ j. DRBD_parallel (λ a. DRBD_series X (L a)) (A j))
      J) =
 Normal
   (product J
      (λ j.
            real
              (1 -
               Normal
                 (product (A j)
                    (λ a.
                          real
                            (1 -
                             Normal
                               (product (L a)
                                  (λ l.
                                        real
                                          (1 -
                                           Normal
                                             (real
                                                (1 -
                                                 prob p
                                                   (X l))))))))))))
```

[PROB_DRBD_series_parallel_series_lem]

```
 ⊢ ∀ p X s L A J.
        (∀ l. l ∈ BIGUNION_o_BIGUNION L A J ⇒ X l ∈ events p) ∧
        indep_sets p (λ i. {X i}) (BIGUNION_o_BIGUNION L A J) ∧
        sets_finite_not_empty2 L A J ⇒
        (prob p
           (DRBD_series
              (λ j. DRBD_parallel (λ a. DRBD_series X (L a)) (A j))
              J) =
         Normal
           (product J
              (λ j.
                    real
                      (1 -
                       Normal
                         (product (A j)
                            (λ a.
                                  real
                                    (1 -
                                     Normal
                                       (product (L a)
```

```
                                        (λ l.
                                            real
                                              (1 -
                                               Normal
                                                 (real
                                                    (1 -
                                                     prob p
                                                       (X l))))))))))))
```

[PROB_DRBD_series_parallel_series_lem1]

```
 ⊢ ∀p X s L A J.
       (∀l. l ∈ BIGUNION_o_BIGUNION L A J ⇒ X l ∈ events p) ∧
       (∀l. l ∈ BIGUNION_o_BIGUNION L A J ⇒ (s l = {l})) ∧
       indep_sets p (λi. {X i}) (BIGUNION_o_BIGUNION L A J) ∧
       sets_finite_not_empty s L A J ⇒
       (prob p
          (DRBD_series
             (λj.
                  DRBD_parallel
                    (λa.
                         DRBD_series (λl. DRBD_parallel X (s l))
                           (L a)) (A j)) J) =
        Normal
          (product J
             (λj.
                  real
                    (1 -
                     Normal
                       (product (A j)
                          (λa.
                               real
                                 (1 -
                                  Normal
                                    (product (L a)
                                       (λl.
                                            real
                                              (1 -
                                               Normal
                                                 (real
                                                    (1 -
                                                     prob p
                                                       (X l))))))))))))
```

[PROB_DRBD_series_parallel_series_rv_red]

```
 ⊢ ∀p X s L A J.
       (∀l.
            l ∈ BIGUNION_o_BIGUNION L A J ⇒
            rv_to_event p X t l ∈ events p) ∧
       indep_sets p (λi. {rv_to_event p X t i})
```

```
     (BIGUNION_o_BIGUNION L A J) ∧
   sets_finite_not_empty2 L A J ⇒
   (prob p
      (DRBD_series
         (λj.
              DRBD_parallel
                (λa. DRBD_series (rv_to_event p X t) (L a))
                (A j)) J) =
    Normal
      (product J
         (λj.
              1 -
              product (A j)
                (λa.
                     1 -
                     product (L a)
                       (λl. 1 - real (1 - Rel p (X l) t))))))
```

[PROB_DRBD_series_rv]

```
⊢ ∀p X t s.
     s ≠ { } ∧ FINITE s ∧
     indep_sets p (λi. {rv_to_event p X t i}) s ⇒
     (prob p (DRBD_series (rv_to_event p X t) s) =
      Normal (product s (λi. real (Rel p (X i) t))))
```

[PROB_R_AFTER]

```
⊢ ∀X Y p fx t.
     rv_gt0_ninfinity [X; Y] ∧ 0 ≤ t ∧ prob_space p ∧
     indep_var p lborel (λs. real (X s)) lborel
       (λs. real (Y s)) ∧
     distributed p lborel (λs. real (X s)) fx ∧
     (∀x. 0 ≤ fx x) ∧ cont_CDF p (λs. real (Y s)) ∧
     measurable_CDF p (λs. real (Y s)) ⇒
     (prob p (DRBD_event p (R_AFTER X Y) t) =
      1 -
      pos_fn_integral lborel
        (λx.
             fx x ×
             (indicator_fn {u | 0 ≤ u ∧ u ≤ t} x ×
              CDF p (λs. real (Y s)) x)))
```

[PROB_R_AND]

```
⊢ ∀p t X Y.
     rv_gt0_ninfinity [X; Y] ∧
     indep_var p lborel (λs. real (X s)) lborel
       (λs. real (Y s)) ⇒
     (prob p (DRBD_event p (R_AND X Y) t) =
      (1 - CDF p (λs. real (X s)) t) ×
      (1 - CDF p (λs. real (Y s)) t))
```

[PROB_R_OR]

$\vdash \forall p\ t\ X\ Y.$
rv_gt0_ninfinity [X; Y] $\wedge$
indep_var p lborel ($\lambda s.$ real (X s)) lborel
($\lambda s.$ real (Y s)) $\Rightarrow$
(prob p (DRBD_event p (R_OR X Y) t) =
1 - CDF p ($\lambda s.$ real (X s)) t $\times$ CDF p ($\lambda s.$ real (Y s)) t)

[R_ABSORB_OR]

$\vdash \forall X\ Y.$ R_OR X (R_AND X Y) = X

[R_AFTER_AFTER]

$\vdash \forall X\ Y\ Z.$
R_AFTER X (R_AFTER Y Z) =
R_OR (R_OR (R_AFTER X Y) (R_AFTER X Z)) (R_AFTER Y Z)

[R_AFTER_D_AFTER]

$\vdash \forall X\ Y\ p\ t.$
DRBD_event p (R_AFTER X Y) t =
p_space p DIFF DFT_event p (D_AND X (D_BEFORE Y X)) t

[R_AFTER_NON_COMM]

$\vdash \forall X\ Y.$ R_OR (R_AFTER X Y) (R_AFTER Y X) = R_NEVER

[R_AFTER_OVER_AND]

$\vdash \forall X\ Y\ Z.$
R_AFTER X (R_AND Y Z) = R_AND (R_AFTER X Y) (R_AFTER X Z)

[R_AFTER_PAND]

$\vdash \forall X\ Y\ p\ t.$
($\forall s.$ ALL_DISTINCT [X s; Y s]) $\Rightarrow$
(DRBD_event p (R_AFTER X Y) t =
p_space p DIFF DFT_event p (P_AND Y X) t)

[R_AND_ALWAYS]

$\vdash \forall X.$ ($\forall s.$ $0 \leq X\ s$) $\Rightarrow$ (R_AND X R_ALWAYS = R_ALWAYS)

[R_AND_ASSOC]

$\vdash \forall X\ Y\ Z.$ R_AND (R_AND X Y) Z = R_AND X (R_AND Y Z)

[R_AND_COMM]

$\vdash \forall X\ Y.$ R_AND X Y = R_AND Y X

[R_AND_ID]

$\vdash \forall X.$ R_AND X X = X

[R_AND_INTER]

$\vdash \forall p\ t\ X\ Y.$
    DRBD_event p (R_AND X Y) t =
    DRBD_event p X t $\cap$ DRBD_event p Y t

[R_AND_NEVER]

$\vdash \forall X.$ R_AND X R_NEVER = X

[R_AND_OVER_OR]

$\vdash \forall X\ Y\ Z.$ R_AND X (R_OR Y Z) = R_OR (R_AND X Y) (R_AND X Z)

[R_INCLUSIVE_AFTER_def2]

$\vdash \forall X\ Y.$
    R_INCLUSIVE_AFTER X Y =
    R_AND (R_AFTER X Y) (R_SIMULT X Y)

[R_LEFT_AFTER_OVER_OR]

$\vdash \forall X\ Y\ Z.$
    R_AFTER X (R_OR Y Z) = R_OR (R_AFTER X Y) (R_AFTER X Z)

[R_OR_ALWAYS]

$\vdash \forall X.$ ($\forall s.$ $0 \leq X\ s$) $\Rightarrow$ (R_OR X R_ALWAYS = X)

[R_OR_ASSOC]

$\vdash \forall X\ Y\ Z.$ R_OR (R_OR X Y) Z = R_OR X (R_OR Y Z)

[R_OR_COMM]

$\vdash \forall X\ Y.$ R_OR X Y = R_OR Y X

[R_OR_ID]

$\vdash \forall X.$ R_OR X X = X

[R_OR_NEVER]

$\vdash \forall X.$ R_OR X R_NEVER = R_NEVER

[R_OR_OVER_AND]

$\vdash \forall X\ Y\ Z.$ R_OR X (R_AND Y Z) = R_AND (R_OR X Y) (R_OR X Z)

[R_OR_UNION]

$\vdash \forall p\ t\ X\ Y.$
    DRBD_event p (R_OR X Y) t =
    DRBD_event p X t $\cup$ DRBD_event p Y t

[R_SIMULT_COMM]

$\vdash \forall X\ Y.$ R_SIMULT X Y = R_SIMULT Y X

[R_WSP_def1]

$\vdash \forall X\_a\ X\_d\ Y.$
`R_WSP` $Y\ X\_a\ X\_d$ = `R_AND` (`R_AFTER` $Y\ X\_d$) (`R_AFTER` $X\_a\ Y$)

[range_from_nat_into]

$\vdash \forall A.$ `countable` $A \wedge A \neq \{\} \Rightarrow$ (`range` (`from_nat_into` A) = A)

[range_from_nat_into_subset]

$\vdash \forall A.\ A \neq \{\} \Rightarrow$ `range` (`from_nat_into` A) $\subseteq A$

[real_mul_real]

$\vdash \forall a\ b.$
$a \neq$ `PosInf` $\wedge\ a \neq$ `NegInf` $\wedge\ b \neq$ `PosInf` $\wedge\ b \neq$ `NegInf` $\Rightarrow$
(`real` $a \times$ `real` b = `real` $(a \times b)$)

[real_normal_real]

$\vdash \forall a.$ `real` (1 - `Normal` (`real` (1 - a))) = 1 - `real` (1 - a)

[Rel_CDF]

$\vdash \forall p\ X\ t.$
`rv_gt0_ninfinity` $[X]\ \wedge$
`random_variable` ($\lambda s.$ `real` $(X\ s)$) p `borel` $\Rightarrow$
(`Rel` $p\ X\ t$ = 1 - `CDF` p ($\lambda s.$ `real` $(X\ s)$) t)

[Rel_R_AFTER]

$\vdash \forall X\ Y\ p\ fx\ t.$
`rv_gt0_ninfinity` $[X;\ Y] \wedge 0 \leq t \wedge$
`indep_var` p `lborel` (`real` $\circ\ X$) `lborel` (`real` $\circ\ Y$) $\wedge$
`distributed` p `lborel` (`real` $\circ\ X$) $fx \wedge (\forall x.\ 0 \leq fx\ x) \wedge$
`cont_CDF` p (`real` $\circ\ Y$) $\wedge$ `measurable_CDF` p (`real` $\circ\ Y$) $\Rightarrow$
(`Rel` p (`R_AFTER` $X\ Y$) t =
1 -
`pos_fn_integral lborel`
($\lambda x.$
$fx\ x\ \times$
(`indicator_fn` $\{u \mid 0 \leq u \wedge u \leq t\}\ x\ \times$
`CDF` p (`real` $\circ\ Y$) x)))

[Rel_R_AND]

$\vdash \forall p\ t\ X\ Y.$
`rv_gt0_ninfinity` $[X;\ Y]\ \wedge$
`indep_var` p `lborel` (`real` $\circ\ X$) `lborel` (`real` $\circ\ Y$) $\Rightarrow$
(`Rel` p (`R_AND` $X\ Y$) t = `Rel` $p\ X\ t\ \times$ `Rel` $p\ Y\ t$)

[Rel_R_CSP]

```
⊢ ∀p X Y f_xay f_y f_cond t.
      rv_gt0_ninfinity [X; Y] ∧ 0 ≤ t ∧
      (∀x y.
           cond_density lborel lborel p (real ∘ X) (real ∘ Y) y
             f_xay f_y f_cond) ∧
      den_gt0_ninfinity f_xay f_y f_cond ⇒
      (Rel p (R_CSP Y X) t =
       1 -
       pos_fn_integral lborel
         (λy.
              indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_y y ×
              pos_fn_integral lborel
                (λx.
                     indicator_fn {w | y < w ∧ w ≤ t} x ×
                     f_cond y x)))
```

[Rel_R_HSP]

```
⊢ ∀p t X Y.
      rv_gt0_ninfinity [X; Y] ∧
      indep_var p lborel (real ∘ X) lborel (real ∘ Y) ⇒
      (Rel p (R_HSP X Y) t =
       1 - (1 - Rel p X t) × (1 - Rel p Y t))
```

[Rel_R_OR]

```
⊢ ∀p t X Y.
      rv_gt0_ninfinity [X; Y] ∧
      indep_var p lborel (real ∘ X) lborel (real ∘ Y) ⇒
      (Rel p (R_OR X Y) t =
       1 - (1 - Rel p X t) × (1 - Rel p Y t))
```

[Rel_R_WSP]

```
⊢ ∀p Y X_a X_d t f_y f_xay f_cond.
      0 ≤ t ∧ (∀s. ALL_DISTINCT [X_a s; X_d s; Y s]) ∧
      DISJOINT_WSP Y X_a X_d t ∧
      rv_gt0_ninfinity [X_a; X_d; Y] ∧
      den_gt0_ninfinity f_xay f_y f_cond ∧
      (∀y.
           cond_density lborel lborel p (real ∘ X_a) (real ∘ Y)
             y f_xay f_y f_cond) ∧
      indep_var p lborel (real ∘ X_d) lborel (real ∘ Y) ∧
      cont_CDF p (real ∘ X_d) ∧ measurable_CDF p (real ∘ X_d) ⇒
      (Rel p (R_WSP Y X_a X_d) t =
       1 -
       (pos_fn_integral lborel
          (λy.
               indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_y y ×
               pos_fn_integral lborel
```

```
                  (λx.
                       indicator_fn {w | y < w ∧ w ≤ t} x ×
                       f_cond y x)) +
         pos_fn_integral lborel
           (λy.
                f_y y ×
                (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
                 CDF p (real ∘ X_d) y))))
```

[RWSP_RCSP]

```
 ⊢ ∀X_a X_d Y.
        (∀s. ALL_DISTINCT [X_a s; Y s]) ∧ (X_d = R_NEVER) ⇒
        (R_WSP Y X_a X_d = R_CSP Y X_a)
```

[RWSP_RHSP]

```
 ⊢ ∀X Y.
        (∀s. ALL_DISTINCT [Y s; X s]) ⇒ (R_WSP Y X X = R_HSP Y X)
```

[sets_finite_not_empty_empty2]

```
 ⊢ ∀L A J.
        sets_finite_not_empty2 L A J ⇔
        sets_finite_not_empty (λl. {l}) L A J
```

[sigma_sets_Inter1]

```
 ⊢ ∀sp st A.
        (∀i. A i ∈ sigma_sets sp st ∧ A i ⊆ sp) ⇒
        BIGINTER {A i | i ∈ 𝒰(:num)} ∈ sigma_sets sp st
```

[sigma_sets_INTER_1]

```
 ⊢ ∀X A B.
        countable B ∧ B ≠ {} ∧
        (∀b. b ∈ B ⇒ b ∈ sigma_sets X A ∧ b ⊆ X) ⇒
        BIGINTER B ∈ sigma_sets X A
```

[sigma_sets_union_1]

```
 ⊢ ∀X A B.
        countable B ∧ (∀b. b ∈ B ⇒ b ∈ sigma_sets X A) ⇒
        BIGUNION B ∈ sigma_sets X A
```

[subset_range_from_nat_into]

```
 ⊢ ∀A. countable A ⇒ A ⊆ range (from_nat_into A)
```

[to_nat_on_finite]

```
 ⊢ ∀A. FINITE A ⇒ BIJ (to_nat_on A) A (count (CARD A))
```

[to_nat_on_infinite]

```
 ⊢ ∀A. countable A ∧ INFINITE A ⇒ BIJ (to_nat_on A) A 𝒰(:num)
```

[UN_from_nat_into]

```
 ⊢ ∀s A.
        s ≠ {} ∧ countable s ⇒
        (BIGUNION {A i | i ∈ s} =
         BIGUNION {A (from_nat_into s i) | i ∈ 𝒰(:num)})
```