1 SENplus Theory

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Parent Theories: DRBD

1.1 Definitions

[DISJOINT3_def]

$\vdash \forall L_1\ L_2\ L_3.$
$\quad \texttt{DISJOINT3}\ L_1\ L_2\ L_3 \iff$
$\quad \texttt{DISJOINT}\ L_1\ L_2 \land \texttt{DISJOINT}\ L_1\ L_3 \land \texttt{DISJOINT}\ L_2\ L_3$

[event_set1_def]

$\vdash \forall X\ i\ Y.\ \texttt{event\_set1}\ (X,i)\ Y = (\lambda j.\ \textbf{if}\ j = i\ \textbf{then}\ X\ \textbf{else}\ Y\ j)$

[event_set2_def]

$\vdash \forall X_1\ i_1\ X_2\ i_2\ Y.$
$\quad \texttt{event\_set2}\ (X_1,i_1)\ (X_2,i_2)\ Y =$
$\quad \texttt{event\_set1}\ X_1\ i_1\ (\texttt{event\_set1}\ X_2\ i_2\ Y)$

[ind_set_def]

$\vdash \forall A.\ \texttt{ind\_set}\ A = (\lambda i.\ \texttt{EL}\ i\ A)$

[SEN_set_req_def]

$\vdash \forall p\ L_1\ L_2\ L\ A\ J\ X.$
$\quad \texttt{SEN\_set\_req}\ p\ L_1\ L_2\ L\ A\ J\ X \iff$
$\quad L_1 \neq \{\,\} \land L_2 \neq \{\,\} \land \texttt{FINITE}\ L_1 \land \texttt{FINITE}\ L_2 \land$
$\quad (\forall l.\ l \in \texttt{BIGUNION\_o\_BIGUNION}\ L\ A\ J \Rightarrow X\ l \in \texttt{events}\ p) \land$
$\quad \texttt{indep\_sets}\ p\ (\lambda i.\ \{X\ i\})\ (\texttt{BIGUNION\_o\_BIGUNION}\ L\ A\ J) \land$
$\quad \texttt{disjoint\_family\_on}\ (\texttt{ind\_set}\ [\{0\};\ L_1;\ L_2;\ \{3\}])$
$\quad\quad \{0;\ 1;\ 2;\ 3\}$

[UNIONL_def]

$\vdash (\texttt{UNIONL}\ [\,] = \{\,\}) \land \forall s\ ss.\ \texttt{UNIONL}\ (s::ss) = s \cup \texttt{UNIONL}\ ss$

1.2 Theorems

[event_set_def]

$\vdash (\forall i_1\ Y\ X_1.\ \texttt{event\_set}\ [(X_1,i_1)]\ Y = \texttt{event\_set1}\ (X_1,i_1)\ Y) \land$
$\quad \forall v_8\ v_7\ i_1\ Y\ X_1.$
$\quad \texttt{event\_set}\ ((X_1,i_1)::v_7::v_8)\ Y =$
$\quad \texttt{event\_set1}\ (X_1,i_1)\ (\texttt{event\_set}\ (v_7::v_8)\ Y)$

[event_set_ind]

$\vdash \forall P.$
$(\forall X_1\ i_1\ Y.\ P\ [(X_1,i_1)]\ Y) \land$
$(\forall X_1\ i_1\ v_7\ v_8\ Y.\ P\ (v_7::v_8)\ Y \Rightarrow P\ ((X_1,i_1)::v_7::v_8)\ Y) \land$
$(\forall v_4.\ P\ []\ v_4) \Rightarrow$
$\forall v\ v_1.\ P\ v\ v_1$

[extreal_sub_sub2]

$\vdash \forall a\ b.$
$a \neq \texttt{PosInf} \land a \neq \texttt{NegInf} \land b \neq \texttt{PosInf} \land b \neq \texttt{NegInf} \Rightarrow$
$(a - (a - b) = b)$

[IN_REST]

$\vdash \forall x\ s.\ x \in \texttt{REST}\ s \iff x \in s \land x \neq \texttt{CHOICE}\ s$

[IN_UNIONL]

$\vdash \forall l\ v.\ v \in \texttt{UNIONL}\ l \iff \exists s.\ \texttt{MEM}\ s\ l \land v \in s$

[normal_real_mul1]

$\vdash \forall a\ b\ c.$
$a \neq \texttt{PosInf} \land a \neq \texttt{NegInf} \land b \neq \texttt{PosInf} \land b \neq \texttt{NegInf} \Rightarrow$
$(\texttt{Normal}\ (\texttt{real}\ a \times \texttt{real}\ b \times c) = a \times b \times \texttt{Normal}\ c)$

[normal_real_mul2]

$\vdash \forall a\ b\ c\ d.$
$a \neq \texttt{PosInf} \land a \neq \texttt{NegInf} \land b \neq \texttt{PosInf} \land b \neq \texttt{NegInf} \Rightarrow$
$(\texttt{Normal}\ (\texttt{real}\ a \times c \times d \times \texttt{real}\ b) =$
$a \times b \times \texttt{Normal}\ (c \times d))$

[normal_real_mul3]

$\vdash \forall a\ b\ c.$
$a \neq \texttt{PosInf} \land a \neq \texttt{NegInf} \land b \neq \texttt{PosInf} \land b \neq \texttt{NegInf} \Rightarrow$
$(\texttt{Normal}\ (\texttt{real}\ a \times c \times \texttt{real}\ b) = a \times b \times \texttt{Normal}\ c)$

[PROB_DRBD_SEN_plus]

$\vdash \forall p\ X\ Y\ Z\ t\ L_1\ L_2\ L\ A\ J.$
$L_1 \neq \{\} \land L_2 \neq \{\} \land \texttt{FINITE}\ L_1 \land \texttt{FINITE}\ L_2 \land$
$(\forall l.$
$l \in \texttt{BIGUNION\_o\_BIGUNION}\ L\ A\ J \Rightarrow$
$\texttt{event\_set}$
$[(\texttt{DRBD\_event}\ p\ Y\ t,0);\ (\texttt{DRBD\_event}\ p\ Z\ t,3)]\ X\ l \in$
$\texttt{events}\ p) \land$
$\texttt{indep\_sets}\ p$
$(\lambda i.$
$\{\texttt{event\_set}$
$[(\texttt{DRBD\_event}\ p\ Y\ t,0);\ (\texttt{DRBD\_event}\ p\ Z\ t,3)]\ X$
$i\})\ (\texttt{BIGUNION\_o\_BIGUNION}\ L\ A\ J) \land$

```
      disjoint_family_on (ind_set [{0}; L1; L2; {3}])
        {0; 1; 2; 3} ∧ (J = {0; 1; 2}) ∧
      (A = ind_set [{0}; {1; 2}; {3}]) ∧
      (L = ind_set [{0}; L1; L2; {3}]) ⇒
      (prob p
         (DRBD_series
            (λ j.
                 DRBD_parallel
                   (λ a.
                        DRBD_series
                          (λ i.
                               event_set
                                 [(DRBD_event p Y t,0);
                                  (DRBD_event p Z t,3)] X i)
                          (L a)) (A j)) J) =
       prob p (DRBD_event p Y t) × prob p (DRBD_event p Z t) ×
       (1 -
        (1 - Normal (product L1 (λ l. real (prob p (X l))))) ×
        (1 - Normal (product L2 (λ l. real (prob p (X l)))))))
```

[PROB_DRBD_SEN_plus_lem1]

```
 ⊢ ∀ p X L1 L2 L A J.
      L1 ≠ { } ∧ L2 ≠ { } ∧ FINITE L1 ∧ FINITE L2 ∧
      (∀ l. l ∈ BIGUNION_o_BIGUNION L A J ⇒ X l ∈ events p) ∧
      indep_sets p (λ i. {X i}) (BIGUNION_o_BIGUNION L A J) ∧
      disjoint_family_on (ind_set [{0}; L1; L2; {3}])
        {0; 1; 2; 3} ∧ (J = {0; 1; 2}) ∧
      (A = ind_set [{0}; {1; 2}; {3}]) ∧
      (L = ind_set [{0}; L1; L2; {3}]) ⇒
      (prob p
         (DRBD_series
            (λ j. DRBD_parallel (λ a. DRBD_series X (L a)) (A j))
            J) =
       prob p (X 0) × prob p (X 3) ×
       (1 -
        (1 - Normal (product L1 (λ l. real (prob p (X l))))) ×
        (1 - Normal (product L2 (λ l. real (prob p (X l)))))))
```

[PROB_DRBD_SEN_plus_rel]

```
 ⊢ ∀ p X Y Ys_a Ys_d Z Zs_a Zs_d t L1 L2 L A J.
      SEN_set_req p L1 L2 (ind_set [{0}; L1; L2; {3}])
        (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
        (event_set
           [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0);
            (DRBD_event p (R_WSP Z Zs_a Zs_d) t,3)]
           (rv_to_event p X t)) ⇒
      (prob p
         (DRBD_series
            (λ j.
```

```
              DRBD_parallel
                (λ a.
                     DRBD_series
                       (λ i.
                            event_set
                              [(DRBD_event p
                                  (R_WSP Y Ys_a Ys_d) t,0);
                               (DRBD_event p
                                  (R_WSP Z Zs_a Zs_d) t,3)]
                              (rv_to_event p X t) i)
                       (ind_set [{0}; L₁; L₂; {3}] a))
                (ind_set [{0}; {1; 2}; {3}] j)) {0; 1; 2}) =
     Rel p (R_WSP Y Ys_a Ys_d) t ×
     Rel p (R_WSP Z Zs_a Zs_d) t ×
     (1 -
      (1 - Normal (product L₁ (λ l. real (Rel p (X l) t)))) ×
      (1 - Normal (product L₂ (λ l. real (Rel p (X l) t))))))
```

[PROB_DRBD_SEN_plus_rel_lem]

```
 ⊢ ∀ p X Y Ys_a Ys_d Z Zs_a Zs_d t L₁ L₂ L A J.
     L₁ ≠ { } ∧ L₂ ≠ { } ∧ FINITE L₁ ∧ FINITE L₂ ∧
     (∀ l.
          l ∈ BIGUNION_o_BIGUNION L A J ⇒
          event_set
            [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0);
             (DRBD_event p (R_WSP Z Zs_a Zs_d) t,3)] X l ∈
          events p) ∧
     indep_sets p
       (λ i.
            {event_set
               [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0);
                (DRBD_event p (R_WSP Z Zs_a Zs_d) t,3)] X i})
       (BIGUNION_o_BIGUNION L A J) ∧
     disjoint_family_on (ind_set [{0}; L₁; L₂; {3}])
       {0; 1; 2; 3} ∧ (J = {0; 1; 2}) ∧
     (A = ind_set [{0}; {1; 2}; {3}]) ∧
     (L = ind_set [{0}; L₁; L₂; {3}]) ⇒
     (prob p
        (DRBD_series
           (λ j.
              DRBD_parallel
                (λ a.
                     DRBD_series
                       (λ i.
                            event_set
                              [(DRBD_event p
                                  (R_WSP Y Ys_a Ys_d) t,0);
                               (DRBD_event p
                                  (R_WSP Z Zs_a Zs_d) t,3)]
```

```
                        X i) (L a)) (A j)) J) =
    Rel p (R_WSP Y Ys_a Ys_d) t ×
    Rel p (R_WSP Z Zs_a Zs_d) t ×
    (1 -
     (1 - Normal (product L₁ (λl. real (prob p (X l)))) ×
     (1 - Normal (product L₂ (λl. real (prob p (X l))))))
```

[real_mul_real]

```
⊢ ∀a b.
    a ≠ PosInf ∧ a ≠ NegInf ∧ b ≠ PosInf ∧ b ≠ NegInf ⇒
    (real a × real b = real (a × b))
```

[Rel_DRBD_SEN_plus]

```
⊢ ∀p X Y Ys_a Ys_d Z Zs_a Zs_d t L₁ L₂.
    SEN_set_req p L₁ L₂ (ind_set [{0}; L₁; L₂; {3}])
      (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
      (event_set
         [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0);
          (DRBD_event p (R_WSP Z Zs_a Zs_d) t,3)]
         (rv_to_event p X t)) ⇒
    (prob p
       (DRBD_event p
          (nR_AND
             (λi.
                  if i = 0 then R_WSP Y Ys_a Ys_d
                  else if i = 1 then
                    R_OR (nR_AND X L₁) (nR_AND X L₂)
                  else R_WSP Z Zs_a Zs_d) {0; 1; 2}) t) =
     Rel p (R_WSP Y Ys_a Ys_d) t ×
     Rel p (R_WSP Z Zs_a Zs_d) t ×
     (1 -
      (1 - Normal (product L₁ (λl. real (Rel p (X l) t)))) ×
      (1 - Normal (product L₂ (λl. real (Rel p (X l) t))))))
```

[Rel_DRBD_SEN_plus1]

```
⊢ ∀p X Y Ys_a Ys_d Z Zs_a Zs_d t L₁ L₂ f_y f_z f_condY f_condZ
    f_ysy f_zsz.
    SEN_set_req p L₁ L₂ (ind_set [{0}; L₁; L₂; {3}])
      (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
      (event_set
         [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0);
          (DRBD_event p (R_WSP Z Zs_a Zs_d) t,3)]
         (rv_to_event p X t)) ∧ prob_space p ∧
    (∀s.
         ALL_DISTINCT
           [Ys_a s; Ys_d s; Y s; Zs_a s; Zs_d s; Z s]) ∧
    DISJOINT_WSP Y Ys_a Ys_d t ∧ DISJOINT_WSP Z Zs_a Zs_d t ∧
    rv_gt0_ninfinity [Ys_a; Ys_d; Y; Zs_a; Zs_d; Z] ∧ 0 ≤ t ∧
```

```
(∀y.
     cond_density lborel lborel p (real ∘ Ys_a)
       (real ∘ Y) y f_ysy f_y f_condY) ∧
den_gt0_ninfinity f_ysy f_y f_condY ∧
indep_var p lborel (real ∘ Ys_d) lborel (real ∘ Y) ∧
cont_CDF p (real ∘ Ys_d) ∧
measurable_CDF p (real ∘ Ys_d) ∧
(∀z.
     cond_density lborel lborel p (real ∘ Zs_a)
       (real ∘ Z) z f_zsz f_z f_condZ) ∧
den_gt0_ninfinity f_zsz f_z f_condZ ∧
indep_var p lborel (real ∘ Zs_d) lborel (real ∘ Z) ∧
cont_CDF p (real ∘ Zs_d) ∧ measurable_CDF p (real ∘ Zs_d) ⇒
(prob p
   (DRBD_event p
      (nR_AND
         (λi.
              if i = 0 then R_WSP Y Ys_a Ys_d
              else if i = 1 then
                R_OR (nR_AND X L₁) (nR_AND X L₂)
              else R_WSP Z Zs_a Zs_d) {0; 1; 2}) t) =
 (1 -
  (pos_fn_integral lborel
     (λy.
          indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_y y ×
          pos_fn_integral lborel
            (λx.
                 indicator_fn {w | y < w ∧ w ≤ t} x ×
                 f_condY y x)) +
   pos_fn_integral lborel
     (λy.
          f_y y ×
          (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
           CDF p (real ∘ Ys_d) y)))) ×
 (1 -
  (pos_fn_integral lborel
     (λy.
          indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_z y ×
          pos_fn_integral lborel
            (λx.
                 indicator_fn {w | y < w ∧ w ≤ t} x ×
                 f_condZ y x)) +
   pos_fn_integral lborel
     (λy.
          f_z y ×
          (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
           CDF p (real ∘ Zs_d) y)))) ×
 (1 -
  (1 - Normal (product L₁ (λl. real (Rel p (X l) t)))) ×
```

```
        (1 - Normal (product L2 (λ l. real (Rel p (X l) t))))))
```

[SEN_nR_AND]

```
⊢ ∀ p X Y Ys_a Ys_d Z Zs_a Zs_d t L1 L2.
      DISJOINT3 {0; 3} L1 L2 ∧ FINITE L1 ∧ FINITE L2 ∧
      L1 ≠ { } ∧ L2 ≠ { } ⇒
      (DRBD_event p
         (nR_AND
            (λ i.
                 if i = 0 then R_WSP Y Ys_a Ys_d
                 else if i = 1 then
                   R_OR (nR_AND X L1) (nR_AND X L2)
                 else R_WSP Z Zs_a Zs_d) {0; 1; 2}) t =
       DRBD_series
         (λ j.
              DRBD_parallel
                (λ a.
                     DRBD_series
                       (λ i.
                            event_set
                              [(DRBD_event p
                                  (R_WSP Y Ys_a Ys_d) t,0);
                               (DRBD_event p
                                  (R_WSP Z Zs_a Zs_d) t,3)]
                              (rv_to_event p X t) i)
                       (ind_set [{0}; L1; L2; {3}] a))
                (ind_set [{0}; {1; 2}; {3}] j)) {0; 1; 2})
```