FORMAL ANALYSIS OF TRAFFIC CONFLICTS SEVERITY USING KEYMAERA

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Abstract

Formal Analysis of Traffic Conflicts Severity using KeYmaera

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Evaluating traffic safety based on crash data is deemed unreliable due to the scarcity of the reported crashes. Furthermore, the process of accumulating a dependable database can take years to be achieved. As an alternative, surrogate safety measures such as Traffic Conflict Techniques (TCTs) are emerging to address many shortcomings of the crash data analysis. However, using data-centric approaches such as simulation to identify traffic conflicts events leading to crashes limits the confidence in road safety assessment. With formal verification, improving traffic safety by accurately analysing traffic conflicts is guaranteed thanks to the mathematical basis and rigorous analysis nature of formal methods.

This thesis aims to complement conventional data-oriented methods for traffic safety assessment with the formal verification of safety properties using the KeYmaera theorem prover based on differential Dynamic Logic (dL). Our main focus is to guarantee the safety of road users through the evaluation of traffic conflicts and providing safer traffic interactions between vehicles. Towards achieving this goal, we propose a set of traffic safety properties based on the combination of different TCTs, i.e., time-to-collision (TTC), space headway (SHW), shockwave speed (SWV), extended delta-V (ΔV) and deceleration rate, along with evasive actions indicators, i.e., jerk profile and yaw rate. The aim of these properties is to formally analyse traffic conflicts in order to determine their severity as well as set accurate traffic conditions to ensure the traffic safety.

In another effort to validate the property linking TTC, SHW and SWV, we use SUMO (Simulation of Urban MObility) on a real-life dataset. Thanks to this traffic simulator, the vehicles dynamics are extracted and a list of TCTs is also provided for every time step. In order to realize a traffic management process, Mathematica is used to interface with SUMO, identify vehicles violating the stated safety property and conduct a speed-adaptation control process to update vehicles' speeds.

In loving memory of my grandfather and my great-grandmother,
To my mother, my father, my brother and my sister

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List of Acronyms

ADAS	Advanced Driver Assistance Systems
API	Application Programming Interface
CACC	Cooperative Adaptive Cruise Control
CS	Conflicting Speed
dL	differential Dynamic Logic
DR	Deceleration Rate
DZ	Dilemma Zone
EBNF	Extended Backus–Naur Form
ET	Encroachment Time
GEH	Geoffrey E. Heavers
IoT	Internet of Things
ITS	Intelligent Transportation Systems
KDE	Kernel Density Estimation
LHS	Left Hand Side
MV	Motorized Vehicles
NHTSA	National Highway Traffic Safety Administration
NMV	Non-Motorized Vehicles
ODE	Ordinary Differential Equation
OIV	Occupant Impact Velocity
PET	Post Encroachment Time
PTW	Powered Two Wheeler
RHS	Right Hand Side
SHW	Space Headway
SMT	Satisfiability Modulo Theories
SPF	Safety Performance Functions
SSAM	Surrogate Safety Assessment Model

SSH	Safety Surrogate Histogram
SSM	Surrogate Safety Measures
SUMO	Simulation of Urban MObility
SWV	Shockwave Speed
TA	Time to Accident
TCT	Traffic Conflict Techniques
TET	Time Exposed TTC
TIT	Time Integrated TTC
TTC	Time-To-Collision
TTD	Travel Time Difference
TTZ	Time-to-Zebra
ΔV	Extended Delta-V

Chapter 1

Introduction

In this chapter, we start by presenting the context and motivation behind this thesis followed by a description of the problem statement and proposed methodology. The chapter is concluded by outlining the main contributions and the organization of the thesis.

1.1 Motivation

In a world of 7.9 billion person with about 1.4 billion motor vehicles in use [73], transportation is of great importance in our daily lives. However, the increasing number of traffic accidents and deaths is a fact that is hard to ignore. Around 1.35 Million road traffic deaths in the world are registered every year [65]. Furthermore, 38,824 lost lives that were registered in 2020 by the National Highway Traffic Safety Administration (NHTSA) [65]. In this context, smart transportation aims to optimize the manipulation of mobility with the aim of making urban transportation more efficient, sustainable and safer [43]. Smart transportation is introduced as an approach that implements a combination of modern technologies into transportation systems, such as internet of things (IoT) along with various techniques and services, e.g., wireless communication, location-based services and computer vision.

On February 14, 2016, Google's self driving car [10] caused its first crash when changing lanes and ending up by putting itself in the path of an oncoming bus. In addition to the two crashes caused by the Zoox vehicle [6] due to the autonomous system misjudging its clearance to parked vehicles, in both instances, and making contact causing minor damage. Hence, there is a dire need to guarantee the safety of vehicles due to the increasing road risks and traffic conflicts faced nowadays. In the context of transportation, a traffic conflict is defined as an interaction between two or multiple vehicles in which one of the vehicles must take an evasive action to avoid a collision. In order to ensure the safety of road users interactions, several *traffic conflict techniques* (TCTs) were introduced to monitor vehicles and their interactions on the road, whether in a normal flow or in a traffic conflict situation. Notable TCTs are Time-To-Collision, space headway and Delta-V, which are applied to determine serious conflicts along with evasive actions-based indicators, e.g., Jerk profile and Yaw Rate. Nevertheless, TCTs and evasive actions-based indicators must be carefully defined and thoroughly analysed taking into consideration their limitations and the intensity of the evasive actions if any is taken. In this thesis, we propose two traffic safety properties that combine a series of traffic conflict indicators along with evasive actions to analyse traffic safety accordingly. This investigation will open new opportunities to combine traffic indicators together and enhance the efficiency of the evaluation. Nevertheless, the resulting properties will only be relevant enough and highly efficient if they are rigorously verified. For instance, using simulation and crash reconstruction methods [17] to verify the efficiency of traffic safety properties based on TCTs does not guarantee the safety of transportation due to the numerous, unforeseen and different traffic conflicts that has to be covered by the verification.

With the increasing level of complexity in transportation, simulation-based methods are no longer sufficient to thoroughly verify a design due to infinite simulation times and poor coverage allowing several bugs to remain undiscovered. Therefore, relying on trustworthy verification tools is the key to achieve sound designs that function correctly. As a complement to simulation, *formal verification* techniques [23] are being investigated and their application in the verification of design correctness is becoming more frequent nowadays. The two most popular formal verification methods to date are *theorem proving* and *model checking*. Model checking [23] is a verification technique applied to verify that a desired property holds for a given state-based model. In order to verify the correctness of the properties, model checkers are applied to verify the state space exhaustively. This verification results in uncovering hidden bugs by covering most of the corner cases. Due to the interaction between their continuous and discrete state transitions, vehicles are modeled as hybrid systems known by their infinite state spaces. The latter cannot be partitioned into finitely relevant regions for deciding reachability [59], which makes model checking incapable of verifying hybrid systems due to their state space explosion problem.

In contrast to model checkers, theorem provers are more suitable to verify hybrid systems thanks to their capability of dealing with infinite domains. Theorem proving [23] is a technique applied to formally verify that a design implementation satisfies its specification. The use of formal logics, e.g., first-order logic and higher order logic, to model systems makes theorem proving capable of analysing large and complex systems. Therefore, theorem proving represents a convenient formal verification method to achieve a sound and accurate verified designs. The aim of this thesis is to apply formal methods, in particular theorem proving, in order to ensure the preservation of TCTs safety properties by drivers or automated vehicles. In particular, we aim to answer the following research questions: *1) What is the impact of the driver's behavior, in a traffic conflict, on traffic safety? 2) What are the implications of shockwaves on traffic safety? 3) How can formal verification help improve traffic safety?.*

1.2 Formal Verification for Transportation: State-of-the-Art

Formal methods and verification tools have been in use in the engineering of safety-critical transport systems for well over 30 years [4]. They have been used in railway, avionics and automative, with the aim of demonstrating, with the highest levels of assurance, the correct functioning of the systems involved. Our interest will be in the application of formal verification on vehicles and traffic system. For instance, Mao et al. [40], applied formal methods to develop a runtime monitoring of a co-operative adaptive cruise control (CACC) system. Toward this goal, they defined temporal specifications for the safe operation of CACC and the results showed that their approach successfully captured specification violation. Pek et al. formally defined lane change maneuver by formalizing traffic rules in [55]. Moreover, the authors conducted the verification of the motion plans of the autonomous vehicle by ensuring a collision-free lane change maneuver which respects the traffic rules. Furthermore, formal verification techniques were also applied to guarantee that an autonomous vehicle will avoid static objects as well as dynamic obstacles on the road [1]. When it comes to the verification of the entirety of the traffic system, Loos et al. [36] developed a distributed car control system and a formal proof that this system is collision-free for arbitrarily many cars, even when new cars enter or leave a multi-lane highway with arbitrarily many lanes.

The work of Mitsch et al., in [47], was one of the early attempts to utilize formal verification tools in the modelling of freeway dynamics. The objective was to ensure that the system correctly calculates the appropriate speed limit and communicates this information to vehicles in certain regions of interest. Differential dynamic logic was used for the formulation and verification of the system specifications in [47]. Seeing the importance of the macroscopic model in planning strategies in allocating resources for implementing optimized and balanced transportation systems, Rashid et al. in [60], opted to formalize some foundation concepts of macroscopic models, namely density, flow rate, mean speed, relative occupancy, and shockwave using the higher-order-logic theorem prover HOL Light. Finally, the authors of [62] provided a formally proved checker of the safe distance rule in order to check if an autonomous vehicle complies with traffic rules.

The studies mentioned above cover the verification of different safety aspects of the vehicle or its interaction with the outer environment. However, none has explored the formalization and verification of Traffic Conflict Techniques (TCT) which serve as surrogate safety measures for traffic interactions.

Table 1.1: Related Work for Formal Verification Classification

Formal Tool / **Reference**	KeYmaera	Isabelle /HOL	HOL Light	Linear Temporal Logic	Breach
[36]	✓				
[47]	✓				
[62]		✓			
[60]			✓		
[9]				✓	
[53]					✓

1.3 Traffic Conflict Techniques

Using crashes to define traffic safety is not proactive and tends to be inaccurate [80]. On one hand, the number of crashes is not enough to base a thorough analysis over the issue. Moreover, it will take years to accumulate a dependable database [80]. This data collection process can be rendered obsolete once it is completed due to the rapidly evolving external factors, such as a change in the infrastructure, new traffic rule safety put in place or an update of the maximum speed in a road section. In this context, several traffic conflict techniques (TCTs) were introduced as a direct evaluation of traffic safety by studying the nature of traffic safety indicators [71, 81]. The TCTs used in the context of this thesis are listed below.

Time-To-Collision

Time-To-Collision (TTC) [24] is a temporal-proximity safety indicator, introduced by Hayward in 1971, in order to identify a potential collision, if no preventative measures are taken. TTC is measured in a real-life traffic conflict situation as the duration of time remained for the collision to occur, if no evasive actions occur and it is considered as an important criterion in traffic conflict techniques. The efficiency of TTC was studied in [14, 28, 74]. In 1993, TTC has been represented as the main indicator when it came to the design of a collision avoidance system [75]. Furthermore, based on the study conducted in [25], it was proven that this indicator impacts the speed of the vehicle one way or another. In fact, a small value of TTC will make the driver brake in order to create a safe distance with the leading vehicle. On the contrary, a longer TTC duration means acceleration for the driver. In [31], the idea was to introduce TTC in a car following model that incorporates the concept of collision sensitivity coefficient. This coefficient is derived from a transformation of TTC, in order to explore its effect on the vehicle's dynamic performance and safety. TTC-based *surrogate safety indicators* [32] emerged not long after TTC. For instance, TTC_{min} which is introduced as the minimum value of TTC, is applied to identify a conflict situation that may lead to a collision. Another derivation from TTC is the time-to-accident (TA) calculated from the moment an evasive action is executed [32]. As another TTC-based surrogate safety indicator, time to zebra (TTZ) [76] is defined as the distance to the zebra crossing divided by the speed at any given moment in time in order to describe the drivers' behavior when approaching the zebra crossing.

Moreover, the time exposed TTC (TET) introduced as the duration of time where TTC is below its defined threshold. Another derivation of TTC is the time integrated TTC (TIT) [46] that uses the integral of the time-to-collision profile of the drivers to express the level of safety. Conventionally, safety-related systems interpreted TTC as an indicator that defines boundaries between a safe situation, a conflict and a serious situation. This interpretation is based on a chosen threshold. In [26], a TTC = 4 seconds indicates that the vehicle in question is in a conflict situation. In the same study, it was shown that a value of 4 to 5 seconds of TTC will result in a number of false positives, i.e., in some situations, for a TTC value between 4 and 5 seconds, a potential collision is indicated. However, in reality, it was a traffic conflict that a normal breaking would have been sufficient to mitigate it safely. Consequently, it was agreed that a TTC threshold of 3 seconds is more convenient to report serious cases with a minimum amount of false alarms [26].

Delta-V

Delta-V [64] is a speed-related traffic conflict indicator that emerged in the 70s to track the velocity difference of the vehicles prior and post-crash [44] denoted as Delta-V (ΔV). In [17], Gabler et al., used crash reconstruction programs, e.g., WinSmash [66] and CRASH3 [51], to estimate the value of Delta-V. Following its emergence, Delta-V became popular for its use in crash-based events. In 1998, Dischinger et al., used Delta-V to study the dynamics and severity of crashes in order to help clinical staff anticipate the development of patient complications and initiate timely prevention strategies [11]. In the same year, Delta-V values and injury levels were exploited to develop chest injury risk curves, in addition to developing a methodology to test the impacts resulting from deactivating airbags [78]. Furthermore, Delta-V was the basis of a statistical prediction model in the URGENCY algorithm to deduce the levels of injuries resulting from a collision [3]. In [16], a comparison between the occupant impact velocity (OIV) and Delta-V was conducted by testing the ability of these severity metrics in predicting injuries resulting from collisions. However, in 2006, Delta-V was applied as a single traffic indicator in the automated conflict analysis algorithms of the surrogate safety assessment model (SSAM), where it proved that it still needs substantial enhancement [64]. Nevertheless, the importance of Delta-V cannot be neglected when it comes to categorizing the reported accidents and determining their level of severity. As a matter of fact, analysing the velocity change reflects the occurrence or the non-occurrence of a crash. However, this will not be efficient in the long run and will not compensate for the fact that Delta-V is unable to predict crashes nor help reduce them, since not all occurring crashes are reported [80].

Extended Delta-V

Extended Delta-V [33] is a more accurate indicator used to estimate the imminence of a crash and the severity should it happen. In order to do so, Extended Delta-V represents the hypothetical value of Delta-V, instead of the true value. Based on its value, Extended Delta-V can be applied to predict crashes. In order to avoid any confusion between the two-related indicators, i.e., Delta-V and Extended Delta-V, we will pinpoint their differences and their similarities. The difference between them lies in the chosen formula for determining the speed prior to the crash, where TTC is integrated in order to define the nearness-to-collision, and multiplied by a deceleration rate to translate the occurrence of traffic conflicts. The severity will be evaluated based on the aggressiveness and remaining time to the collision. However, the common part is that Extended Delta-V calculation still abides by the defined formula of Delta-V [64]. We should note that Extended Delta-V is equal to Delta-V once the crash has occurred.

Space Headway

Space Headway (SHW) [27] is defined as the physical distance separating two consecutive vehicles. The value of SHW is determined as the difference between the position of the front of the leading vehicle and the position of the front of the following vehicle as shown by Figure 1.1. SHW is also defined as the inverse of the traffic density indicator (k) where k is the average number of vehicles occupying one mile or one kilometer of road space. As a spatial proximity indicator, SHW was used in [67] to describe a safe traffic flow where the shockwave occurrence is improbable for a series of equal space headways over a platooning of vehicles. In [19], Ghasemi et al. presented a state-of-the-art approach for the evaluation of the statistical parameters of the headway by utilizing the reliability analysis. The study in [63] attempted to estimate the average space headway using a model based approach with special reference to congestion prediction for intelligent transportation systems (ITS) applications.

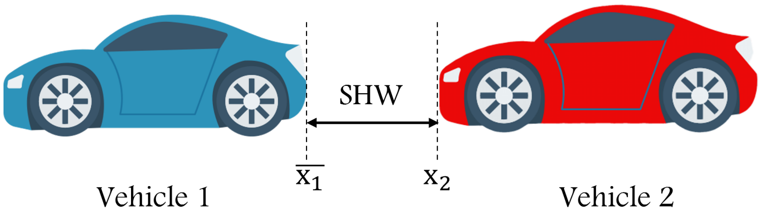

Figure 1.1: Illustration of Space Headway (SHW) Computation

Deceleration Rate

Deceleration Rate (DR) [42] is commonly known as the opposite of acceleration. While acceleration reflects the rate at which a vehicle speeds up, the deceleration is defined as the opposite by reflecting the rate at which the vehicle slows down. Considered as a traffic conflict indicator, DR was involved in different works, such as the work of [18], where the focus was on extracting the initial DR among other surrogate safety measures from the simulation model in order to identify conflict situations based on their values. In [5], the acceleration and deceleration behavior for different vehicle types was studied in order to determine the ideal duration of yellow light at intersections. Furthermore, Bokare et al. based their analysis on the initial DR as a useful measure to identify the severity of the potential conflict event. In [77], the authors studied the deceleration behaviors for drivers of passenger cars at stop sign–controlled intersections. This study deduced the importance of approach speeds at impacting drivers' deceleration behavior. Among other results, the authors concluded that drivers approaching at high speed normally have higher deceleration rates at the beginning of their deceleration.

Shockwaves

Shockwaves (SWV) [61] are defined as byproducts of traffic congestion and queues. Furthermore, they represent the transition zones between two traffic states that move through a traffic environment. In the literature, it is used as an existing event where the analysis takes place after its occurrence. In the work done by Essa et al. [13], the developed conflict-based Safety Performance Functions (SPF) aimed to define the relation between rear-end collisions and explanatory variables such as shockwave speed. The main goal was to prove that rear end collisions mainly take place in shockwave areas where dynamic traffic variables, such as shockwave speed and maximum queue length are used as indicators. In [8], the main focus of the authors was on investigating the frequency of rear-end crashes in congested freeways in the presence of a downstream shockwave. The latter was used as an environmental situation that can be a factor for rear-end crashes. In the work of Machiani et al. [38], a novel surrogate safety measure called safety surrogate histogram (SSH) was developed by taking into consideration the frequency of Dilemma Zone (DZ)-related crashes. The concept of SSH is related to the behavior of traffic passing through the forming shockwave at the intersection without providing the characteristics nor the parameters of a shockwave. However, in [67] the notion of shockwave was well explored and its mathematical definition was provided both in the macroscopic and microscopic levels. The goal was to study the shockwave and its speed to establish the conditions that must be preserved by vehicles in order to avoid conflicts in shockwaves areas. However, none of the mentioned work above studied the occurence of shockwaves by evaluating the computed shockwave speed compared to a pre-defined threshold.

1.4 Evasive Actions Indicators

Traffic conflicts are mainly analysed through the computation of conventional traffic conflict techniques, such as TTC, PET and SHW. As highlighted by Table 1.2, these severity metrics are applicable under the assumption that the studied conflict results from spatial and temporal proximity interactions. However, this assumption becomes invalid in situations of close interactions between vehicles, e.g., congested areas and existence of shockwaves, where braking is expected to happen frequently. As an alternative, evasive actions based indicators are exploited to perform a more accurate traffic conflict analysis [69]. In this thesis, we investigate two evasive actions based indicators, i.e., jerk profile and yaw rate, as described below.

Jerk Profile

The jerk profile [41] represents the temporal dynamics of the acceleration of a vehicle. It is computed as the derivative of the acceleration. The authors in [48] classified the drivers driving style based on the extracted jerk profile of the driver in question. An algorithm was developed for this purpose to extract the jerk features from the current vehicle speed and classifies the current driver style into three classes, calm, normal and aggressive, by comparing the extracted jerk feature with the statistics of the driver styles. The authors in [2] analysed the number of critical jerks extracted from a dataset of 166 private cars. This number was compared to self reported accidents in order to verify the accuracy of the jerk and its correlation with critical accident situations. Through this work, they confirmed that the jerk profile makes it possible to identify safety critical driving behavior that may lead to accidents. In [79], the authors' work focused on proving the validity of the jerk profile as a traffic conflict indicator as well as investigating its efficiency in identifying potential conflicts. This quest was met by their findings that confirmed the shortcomings of the deceleration rate as conflict indicator as well as the success of the jerk profile indicator in identifying conflicts that went undetected by conventional conflict indicators.

Yaw Rate

The yaw rate [22] is a conventional evasive action describing the intensity of the swerving of a vehicle. In the work of [70], the yaw rate was compared to temporal proximity indicators, e.g., TTC to evaluate their ability in identifying motorcycle conflicts in highly mixed and less-organized traffic environments. Based on the results of the case study [70], the evasive actions indicators, e.g., yaw rate, proved to be highly efficient than temporal proximity indicators. The authors in [21] evaluated the severity of powered two-wheeler (PTW) conflicts by comparing the time proximity (TTC) indicator and evasive action-based, e.g., yaw rate indicators. The work done in [35] aims to model lateral interaction between motorized vehicles (MV) and non-motorized vehicles (NMV) in mixed traffic roads. Based on the findings of [35], the yaw rate is considered as a significant factor affecting the critical lateral distance significantly. In fact, the authors of [35] noted that the probability of the lateral interaction gets lower at higher NMV and MV yaw rates.

Table 1.2 represents the classification of the above described TCTs and evasive actions based-indicators according to the type of the indicator.

Table 1.2: Classification of Traffic Conflict Techniques

Type of Indicators	**Traffic Safety Measures**
Temporal Proximity	TTC, PET, TA, TTZ, TET, TIT, Gap time, ET, Deceleration to safety time, Headway, Time advantage, TTD, Breaking time
Spatial Proximity	Space Headway, Remaining distance to potential point of collision, Proportion of stopping distance, range or range rate, lateral distance to departure
Severity of the Conflict	Deceleration rate, Jerk profile, Yaw rate
Impact of the Crash	Delta-V, Extended Delta-V

1.5 Problem Statements

The dire need to improve transportation safety made it essential to study traffic conflicts. Based on the work done by Zheng et al. in [80], two types of traffic conflicts were introduced as follows 1) Traffic conflicts marked by evasive actions; and 2) Temporal (and (or) spatial) proximity based traffic conflicts. The two types are characterized by either a set of actions or a set of indicators to identify a traffic situation as a traffic conflict. The use of Surrogate Safety Measures (SSM), more specifically Traffic Conflict Techniques (TCT), proved to be beneficial when it comes to identifying traffic conflict situations and analysing them. Nonetheless, the use of these traffic indicators has shown certain limitations when it comes to identifying conflicts representing near-misses to serious conflicts. In the sequel, we describe alternatives as a remedy to this problem:

1. The use of a sole traffic indicator proved to be insufficient to conduct an accurate safety traffic analysis by being incapable of capturing all aspects of the traffic situation accurately [32]. Furthermore, the convenient traffic indicator should be employed to provide an accurate evaluation.

2. The use of a combination of TCTs to better analyse traffic conflicts will only reflect the current dynamics of the vehicle, i.e., position, speed, headway, due to the actions taken by the driver. The TCTs based-analysis is sufficient to a certain extent to help improve traffic safety, however its efficiency is not always guaranteed in every situation.

3. When it comes to testing the validity of TCTs, simulation-based methods were applied. For example, usage of crash-simulation software to reconstruct the accident allows analysis and extraction of the key factors [44]. However, with the increasing complexity of transportation systems, simulation represents quite a challenge to cover all possible cases.

1.6 Proposed Methodology

The aim of this thesis is to enhance transportation safety by employing traffic techniques in the safety analysis of traffic conflicts. Working on improving traffic safety can be achieved on two scales [60]: 1) macroscopic level; and 2) microscopic level. The macroscopic model allows to oversee of the traffic behavior and interactions while abstracting the minuscule details by not focus on the individuality of a vehicles. The main focus of this model is on the complete scene which consist of vehicles and traffic events. On a smaller scale, the microscopic model captures each vehicles' dynamics as well as the drivers' actions which facilitates the evaluation of the traffic state [67]. This level of abstraction offers a wider range of surrogate safety indicators, more specifically TCTs, to accurately conduct a complete safety analysis in a traffic conflict. Consequently, we opt to conduct our study at the microscopic model. Towards achieving this goal, the formal analysis of certain TCTs and evasive actions in this work is conducted along with the formation of TCTs based safety transportation properties. To this end, formal verification is exploited in order to verify the non-violation of the defined properties and their preservation in traffic conflicts.

In order to overcome the stated limitations, we propose Time-To-Collision (TTC), Extended Delta-V (ΔV), Shockwave Speed (SWV), Space Headway (SHW) and Deceleration Rate (DR) as the main TCTs employed in this work, along with the evasive actions-based indicators; Jerk Profile and Yaw Rate. Figure 1.2 depicts the proposed methodology, where we start by formalizing the stated TCTs and evasive actions in dL in order to define two traffic safety properties. These properties are later verified for different collision scenarios and traffic events, i.e., shockwaves. The aim of this work is to conduct a formal safety analysis based on defined thresholds of the described TCTs and evasive actions-based indicators, in order to determine the consequences of violating the traffic safety properties.

As Figure 1.2 depicts, we define the first traffic safety property to establish a relationship between Time-To-Collision and Extended Delta-V in order to conduct a formal analysis of crash severity depending on the driver behavior during the traffic conflict. This study is based on the analysis of traffic conflict indicators and their violation or not of the predefined thresholds. Moreover, we introduce the Jerk Profile and Yaw Rate as the evasive actions carried by the driver, before or during the conflict, to study their intensity and determine their efficiency. Furthermore, we study the traffic safety property in common traffic collision scenarios, such as rear-end and right-side conflicting events in order to conduct a crash severity analysis. We determine the crash severity by evaluating a sequence of evasive actions that should be taken to avoid the traffic conflict. However, if the right action is not executed or not sufficient then we evaluate the crash severity based on the vehicles' velocities and whether or not they avoid the traffic conflict.

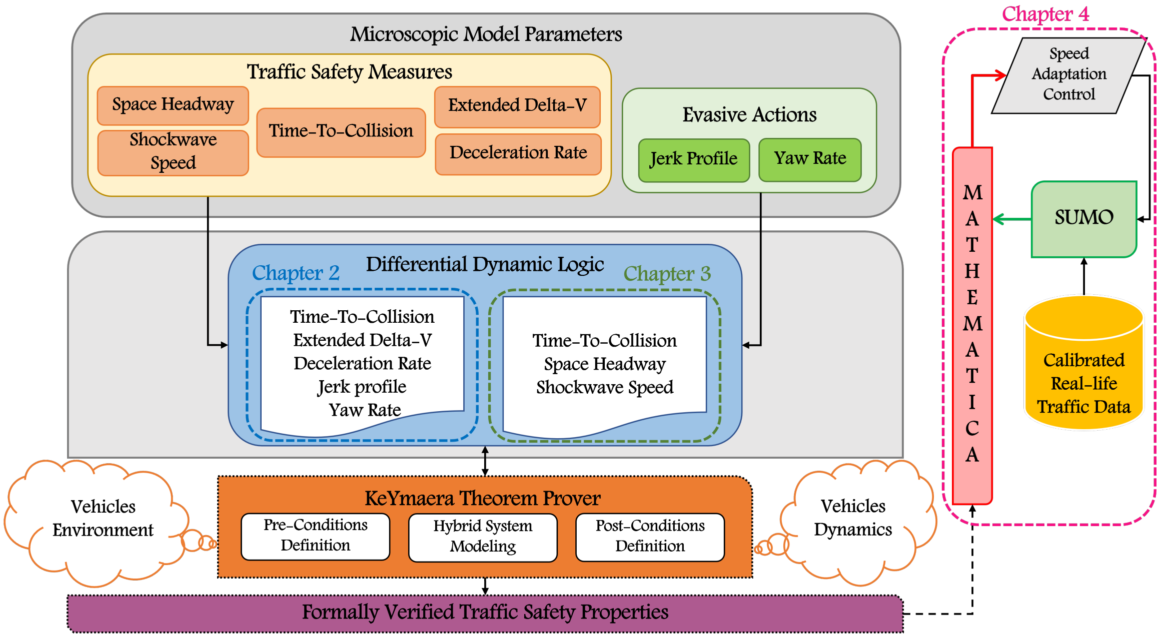

Figure 1.2: Proposed Methodology

The second traffic safety property of interest includes TTC, space headway and shockwave speed in order to study the impact of traffic events, such as shockwaves, on the mentioned traffic safety indicators. The aim of this property is to formalize the relationship between shockwaves and TTC and monitor the impact of shockwave on TTC and vice versa in a car following model [20]. In this model, the response of a vehicle in a traffic flow strongly depends on the behavior of its predecessor. We validate the conducted formal analysis of the TTC-Shockwave property over a real-life traffic dataset using the traffic simulation tool SUMO [37] in order to identify the cases violating the property and their ramifications in the traffic flow.

Finally, the reduction of traffic conflicts will be achieved with the implementation of an adaptive traffic management system. As Figure 1.2 depicts, the speed adaptation process established through the interfacing of a computer algebra system Mathematica [39] and SUMO with the aim to control the speed of violating vehicles. This control consists of assigning convenient speed values to the vehicles in question in order to ensure their adaptation to the upcoming traffic conflict.

In order to formalize the above traffic safety properties, we express them in Differential Dynamic Logic (dL) [57], which is a natural specification and verification logic for hybrid systems. Thanks to its proof calculus, dL is used to describe/verify correctness properties for hybrid systems. However, without a formal verification tool the formalization will only remain an unfinished work. Due to its hybrid nature enabling the verification of hybrid systems, e.g., vehicles, we propose KeYmaera [59] as the main theorem prover for our safety properties.

As depicted by Figure 1.2, the KeYmaera theorem prover requires a three-step formalization process. We start by defining the preconditions needed for variables to be in their respective ranges guaranteeing an accurate translation of the property. Next, the core of the process is dedicated to the code of the property where different situations are covered based on a sequence of conditions representing the post-conditions for the system parameters. Lastly, a mathematical representation of the hybrid system is given by an ordinary differential equation (ODE) modeling the system dynamics, e.g., position, velocity and acceleration. Once the formalization is done, we use KeYmaera to verify the correctness of the traffic safety properties.

1.7 Thesis Contributions

This work aims to improve the safety of transportation by studying essential safety properties combining surrogate safety measures (SSM) defining relevant TCT safety properties and using formal methods to verify them. The contributions of the thesis can be summarized as follows:

- We use traffic safety measures at the microscopic model, in addition to the jerk profile and yaw rate, to define two traffic safety properties. Moreover, we perform an analysis of the traffic flow in order to identify traffic conflicts that might lead to accidents.
- We apply Differential Dynamic Logic (dL) as the specification and verification logic to formalize the traffic safety properties. Furthermore, we conduct the verification of the two safety properties using the KeYmaera theorem prover.
- We perform the validation of the traffic safety property linking Time-To-Collision (TTC), Space Headway (SHW) and Shockwaves (SWV) using the traffic simulation tool SUMO over a real-life traffic dataset of a highway situated in Florida, USA.
- We establish an adaptive traffic management system, thanks to the integration of SUMO with the computer algebra system Mathematica. This system consists of a speed adaptation process aiming to adjust vehicles' speeds to adapt to the traffic conflict.

1.8 Thesis Organization

The rest of this thesis is organized as follows: in Chapter 2, we formalize the traffic conflict indicators, i.e., Time-To-Collision (TTC), Delta-V and Extended Delta-V along with a set of evasive actions, i.e., Jerk profile and Yaw rate, using dL in KeYmaera. This formalization is a means to determine the severity levels of crashes. The identification of crashes occurrences and the determination of their severity is also demonstrated. The conducted formal analysis of those surrogate indicators is applied to common traffic collision scenarios such as rear-end and head-on conflicting events. In Chapter 3, a safety transportation property combining TTC, SHW and

SWV is presented. The nature of the relation is defined to be a bidirectional relation. Subsequently, the formalization as well as the verification of the property is conducted to prove its correctness. Followed by its validation over a real-life dataset. In Chapter 4, a case study is conducted in order to evaluate the efficiency of the TTC-SWV traffic safety property using a real-life traffic dataset by implementing an adaptive traffic management system. This system consists on executing a speed adaptation process updating vehicles' speeds in order to reduce traffic conflicts. The thesis is concluded by Chapter 5 providing a summary of the proposed research and an outline of future work.

Chapter 2

Formal Analysis of Crash Severity

This chapter delivers a formal analysis of crashes severity conducted through the analysis of a traffic safety property. We start by providing a preliminary of the employed TCTs, followed by a description of the proposed methodology of the TTC, SHW and SWV safety property. Lastly, we describe the formalization and verification of the proposed safety property using the automated theorem prover, KeYmaera, in detail.

2.1 Preliminaries

2.1.1 Mathematical Modeling of TCTs

Time To Collision

TTC is defined as the time required for two vehicles to collide if they continue at their present speed and on the same path [24]. TTC can be computed for different traffic interactions. In our case, we define it in a rear-end, head on and right-side collision situations as follows:

- **Rear-end collision:** TTC is measured between two consecutive vehicles from the rear bumper of the leading vehicle to the front bumper of the following vehicle. The mathematical formula to compute TTC for two consecutive vehicles is defined in Equation 2.1 [46].

$$TTC = \frac{x_1 - x_2 - L_1}{v_2 - v_1}, \qquad v_2 > v_1 \tag{2.1}$$

where vehicle 1 and vehicle 2 are the leading and following vehicles, respectively. x_1, x_2, v_1 and v_2 are the positions and velocities of vehicles 1 and 2, respectively, and L is the length of vehicle 1.

- **Head-on collision:** In this case, the formula of TTC is modified as [34]:

$$TTC = \frac{x_1 - x_2}{v_1 + v_2} \tag{2.2}$$

where x_1, x_2, v_1 and v_2 are the positions and velocities of vehicle 1 and 2, respectively.

- **Right-side collision:** In this case, the definition of TTC is updated based on the time instant of the front of vehicle 1 and vehicle 2, i.e., t_{f1} and t_{f2}, respectively, the mathematical definition is presented as follows [34]:

$$t_{f1} < t_{f2} < t_{r1} \tag{2.3}$$

$$t_{f2} < t_{f1} < t_{r2} \tag{2.4}$$

If Equation (2.3) is satisfied, then

$$TTC = \frac{d_2}{v_2} \tag{2.5}$$

otherwise:

$$TTC = \frac{d_1}{v_1} \tag{2.6}$$

where v_1 and v_2 represent the vehicles' velocities and d_1 and d_2 are defined as the distances separating vehicle 1 and 2, respectively, from the region S as shown in Figure 2.1.

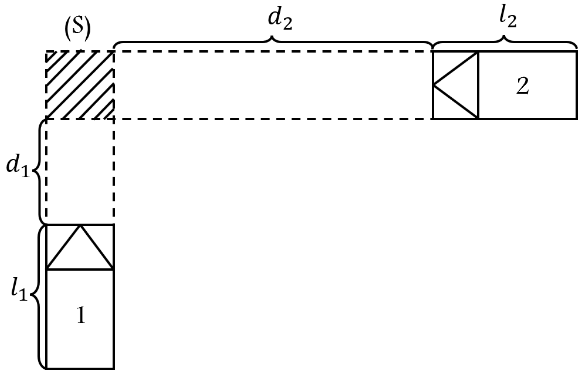

Figure 2.1: Collision Course for a Right-Side interaction [34]

Extended Delta-V

Extended Delta-V is a speed-related indicator describing the speed reduction rate of vehicles due to an unexpected event, e.g., conflict, collision [33]. It is used as an informative measure about the possibility of a crash occurrence and whether or not a preventive action was taken. The value of this indicator foresees the severity of the collision should it happen. For this, Extended Delta-V represents the theoretical value of Delta-V if the taken evasive action was successful. In the case where the collision takes place, the value of Extended Delta-V converges to the true value of Delta-V. Whereas, Extended Delta-V abides by the same general rule to determine the value of Delta-V, given by Equation 2.7 to determine its value. The specificity of this indicator compared to the classical Delta-V lies in the vehicle's speed definition prior to a conflict. Based on its mathematical definition in Equation 2.8, this indicator incorporates braking as an evasive measure linked to a temporal indicator. In a traffic conflict, the chosen temporal indicator within a collision course is Time-To-Collision (TTC). Equation 2.8 presents the initial speed formula of the vehicle used to determine Extended Delta-V during a normal traffic flow, TTC was integrated in order to define the nearness-to-collision, multiplied by a deceleration rate to translate the evasive action needed in order to avoid the crash. The severity will be evaluated based on the aggressiveness and remaining time to the collision.

$$\Delta V = V_{post} - V \tag{2.7}$$

$$V = v_0 - a * t \tag{2.8}$$

Under the assumption that it is an *inelastic collision* and in the case of two vehicles colliding, those vehicles will stick together after the crash, i.e., $\mathrm{V}_{1post} = \mathrm{V}_{2post} = \mathrm{V}_{post}$. Mathematically, an inelastic collision [64] is translated by Equation 2.9:

$$m_1 * v_1 + m_2 * v_2 = (m_1 + m_2) * V_{post} \tag{2.9}$$

V_{post} is deduced as shown in Equation 2.10

$$V_{post} = \frac{m_1 * v_1 + m_2 * v_2}{m_1 + m_2} \tag{2.10}$$

Jerk Profile

Based on the work done in [79], braking as an evasive action was deemed to be insufficient seeing that it may not be successfully executed in certain situations in order to distinguish conflicts from collisions. Furthermore, the deceleration profile on its own is not capable of capturing the severity of vehicles' interactions. In [2], it was shown that the acceleration profiles studied in normal situations are rather similar to the captured profiles in a conflict situation. These findings fueled researchers to look for a complementary indicator of traffic conflicts. Observing acceleration rate of change going from light braking to sudden and intense braking can be detected using the Jerk profile. This indicator is defined as the temporal rate of variation of the acceleration profile as shown in the mathematical definition given by Equation 2.11, where a is the vehicle's acceleration.

$$J = \frac{\mathrm{d}a}{\mathrm{d}t} \tag{2.11}$$

By analysing the jerk profile, we are able to distinguish traffic conflict situations from normal to near misses situations. For instance, a traffic conflict is characterized by a strong negative value of the jerk, where the highest value computed by the jerk is found to be -15 $\mathrm{m/s^3}$ seeing that any greater value will be considered mechanically unfeasible [79]. As for a situation where normal braking is executed, the highest value of jerk computed is at -8 $\mathrm{m/s^3}$. Based on the findings of multiple researchers, such as [54] and [2], a threshold value of the jerk profile is defined in Equation 2.12, to be equal to -9.82 m/s^3, as an indicator of safety-critical driving behavior.

$$J_{critical} = -9.82m/s^3 \tag{2.12}$$

Starting from this defined threshold, the situation at hand reflects an endangering traffic conflict at which the driver should react efficiently and fast.

Yaw Rate

An unsuccessful execution of the evasive action might have dire consequences in certain cases. Braking, as an evasive action, is not always sufficient to avoid a crash. Depending on the situation he is in, if the driver does not conduct the right deceleration rate, the accident is imminent. Based on the computed value of the jerk profile, we are able to determine if the evasive action was successful or not. However, this does not mean that the only type of evasive action that can be chosen is braking. Changing the vehicle's trajectory suddenly, either to the right or left, and in a short amount of time is called swerving in a conflict situation. As an evasive action, swerving is executed once the vehicle is found in a conflict situation by varying the heading angle of the vehicle in a chosen direction. The yaw rate is used as the indicator to quantify swerving, this profile is used to describe the change of the heading angle and analyse its value in a short period of time. In fact, the yaw rotation is the movement around the yaw axis of a body that changes direction to the left or right of its direction of motion. As depicted by Figure 2.2, the yaw rate is the angular velocity of the rotation around the vertical axis (z-axis) or the rate of change of the heading angle. Mathematically, the yaw rate is defined in Equation 2.13 as follows, where θ is the vehicle's heading angle [68].

$$r(t) = \frac{\mathrm{d}\theta}{\mathrm{d}t} \tag{2.13}$$

The range of the yaw rate is defined based on the safety of the action after its execution, in this case a range of [0, 0.785] rad/s [7] reflects a swerving between [0, 45] degrees. A value of the yaw rate in this range describes an abrupt swerving due to the occurrence of a traffic conflict. The computed yaw rate describes a successful execution of the evasive action while maintaining control of the vehicle. As a result, we define the bounds of the yaw rate indicator in Equation 2.14.

$$0 < r \leq 0.785 \tag{2.14}$$

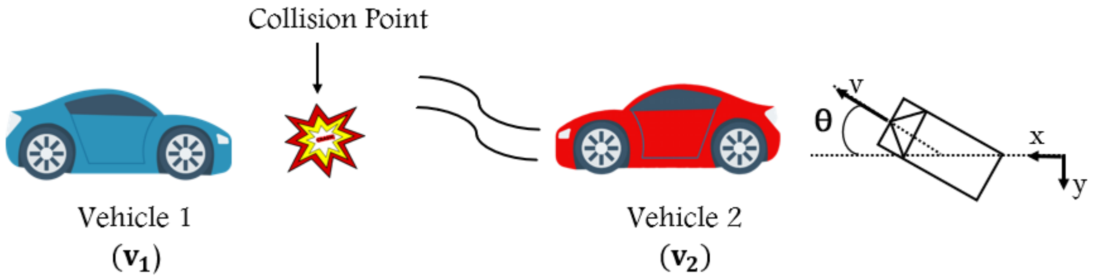

Figure 2.2: Description of Swerving as an Evasive Action

2.1.2 KeYmaera: An Automated Theorem Prover

Formal Verification has been gaining reputation due to its involvement when it comes to safety critical systems, such as vehicles, that are growing significantly in complexity. In many applications, certain system states, e.g., the positions of vehicles that are changing continually due to differential equations are affected by the decisions made by discrete controllers [59]. This interaction between continuous and discrete state transitions defines systems that are mathematically modeled using hybrid systems. As a deductive verification tool that deals with hybrid systems, KeYmaera handles hybrid systems' arithmetic by using real quantifier elimination. In the handling of differential equations of continuous evolutions, KeYmaera applies symbolic computations in computer algebra systems [59]. As an automated and interactive theorem prover for a natural specification and verification logic for hybrid systems, KeYmaera supports differential dynamic logic (dL), i.e., a real-valued first-order dynamic logic for hybrid programs. Furthermore, dL is introduced as a program notation for hybrid automata [59]. In several real-life applications, the proof construction can be completely automatic, e.g., for proving collision avoidance of trains or aircrafts. KeYmaera theorem prover is distinguished by an implemented plug-in architecture for integrating multiple instances of decision procedures for different fields of arithmetic handling. Furthermore, its interfacing with computer algebra systems, such as

Mathematica [39], allows the integration of arithmetics simplification and real quantifier elimination support. Moreover, symbolic solutions of differential equations, that can be used for handling continuous dynamics, can be obtained from external solvers such as the Java math library Orbital [59].

Hybrid systems are mathematical models for systems with interacting continuous and discrete state transitions. They represent an extension of discrete regular programs by continuous evolutions. The operational behavior of hybrid systems can be described using hybrid automata. In order to verify systems by symbolic decomposition, compositional semantics are integrated in the program notation for hybrid automata. Moreover, hybrid automata can be embedded into hybrid programs. An overview of the syntax and informal semantics of hybrid programs is given in Table 2.1, where F is a formula of first-order real arithmetic.

For state characterization of hybrid systems, KeYmaera's specification and verification logic is founded on first-order logic over real arithmetic. In order to express correctness statements about hybrid systems, this foundation is extended with parameterized modal operators $[\alpha]$ and $<\alpha>$, for each hybrid program α. Furthermore, the resulting specification and verification logic is called differential dynamic logic *dL* [59]. *dL* represents the most fundamental logic for dynamic systems that is used to verify correctness properties for hybrid systems. The modal operators refer to states reachable by the hybrid program α and can be placed in front of any formula. The formula $[\alpha]\phi$ expresses that all states reachable by the hybrid program α satisfy formula ϕ (safety). The formula $<\alpha>\ \phi$ represents the existence of a state reachable by the hybrid program α that satisfies formula ϕ [59].

dL is a single language that integrates operational system models and formulas. The dL formulas are generated by the following Extended Backus–Naur Form (EBNF) grammar, where $\sim\ \in \{<, \leq, =, \geq, >\}$ and θ_1 and θ_2 are arithmetic expressions in +, -, ., / over the reals [59].

$$\phi ::= \theta_1 \sim \theta_2 \mid \neg\phi \mid \phi \wedge \psi \mid \forall x\phi \mid \exists x\phi \mid [\alpha]\phi \mid < \alpha > \phi$$

Table 2.1: Statements of Hybrid Programs [59]

Statement	**Effect**
$\alpha;\beta$	sequential composition, first performs α and then β afterwards
$\alpha \cup \beta$	nondeterministic choice, following either α or β
α^*	nondeterministic repetition, repeating α $n \geq 0$ times
$x := \theta$	discrete assignment of the value of term θ to variable x (jump)
$x := *$	nondeterministic assignment of an arbitrary real number to x
$x'_1 = \theta_1,..., x'_n = \theta_n,\ F$	continuous evolution of x'_i along differential equation system $x'_i = \theta_i$, restricted to a maximum domain or an invariant region F
$?F$	check if formula F holds at current state, abort otherwise
if (F) then α	perform α if F holds, do nothing otherwise
if (F) then α else β	perform α if F holds, perform β otherwise

2.2 Crash Severity Analysis Methodology

An overview of the proposed methodology to formally analyse crashes severity is provided in Figure 2.3. In order to determine the severity level of a crash, we start by introducing Time-To-Collision (TTC), Deceleration Rate (DR) and Extended Delta-V (ΔV) as the applied safety traffic indicators in this analysis. These indicators are evaluated according to certain pre-defined thresholds. The non-satisfaction of these constraints leads to certain outcomes allowing us to determine the severity level of crashes. In addition, we take into consideration the presence of evasive actions and their efficiency in avoiding the accident. In this study, different speed intervals are considered to accurately define the level of crash severity. Initially, we start by defining the formulas of traffic conflict indicators based on each collision scenario and formalizing them. Subsequently, we derive the property specifying the severity levels in each crash scenario. The first step in defining this property is to introduce its pre-conditions followed by the hybrid system modeling using an ordinary differential equation (ODE) due to the continuous evolution of the system. The next step is to define the severity levels of crashes and stop them from happening by decelerating when deemed necessary. Lastly, the post-conditions are introduced in order to describe every speed interval and its association with a deceleration variable.

The deceleration-speed association guarantees the property's satisfaction by avoiding the crash. However, the situation differs due to the variance in the drivers behavior. This uncontrolled parameter leads us to assume that in some cases the deceleration rate will not be sufficient to avoid the crash. This is where the evasive actions come into play to provide an accurate analysis of the situation due to their capability of reflecting the intensity of the action in the moment of the conflict. By proving that the formalized property holds for the given conditions and assumption, we identify different severity levels for crashes that are inevitable, e.g., no evasive action. We achieve this by monitoring the variation of the value of Extended Delta-V and the speed of the two involved vehicles entering the traffic conflict. Once the involved vehicles are found in a traffic conflict, the interpretation of the two indicators based on the executed deceleration determines the severity level of the crash should it happen. For this, in Table 2.2, we propose a list of the speed intervals of the involved vehicles in the traffic conflict that range from 0 to more than 80 km/h. For every speed interval, we assign a deceleration rate to be executed in case of a traffic conflict. However, this deceleration rate is only an estimation since we cannot determine the exact intensity of the braking without further information to take into account the surrounding conditions and environment of the conflict. We propose the following decelerations, as depicts Table 2.2, where b_1, b_2, b_3, and B are positive variables and $b < b_1 < b_2 < b_3 < B$. In order to formalize the TCTs (TTC and Extended Delta-V) and the evasive actions indicators (Jerk Profile and Yaw Rate), and verify the property combining both TCTs and evasive actions, we use the KeYmaera theorem prover [56].

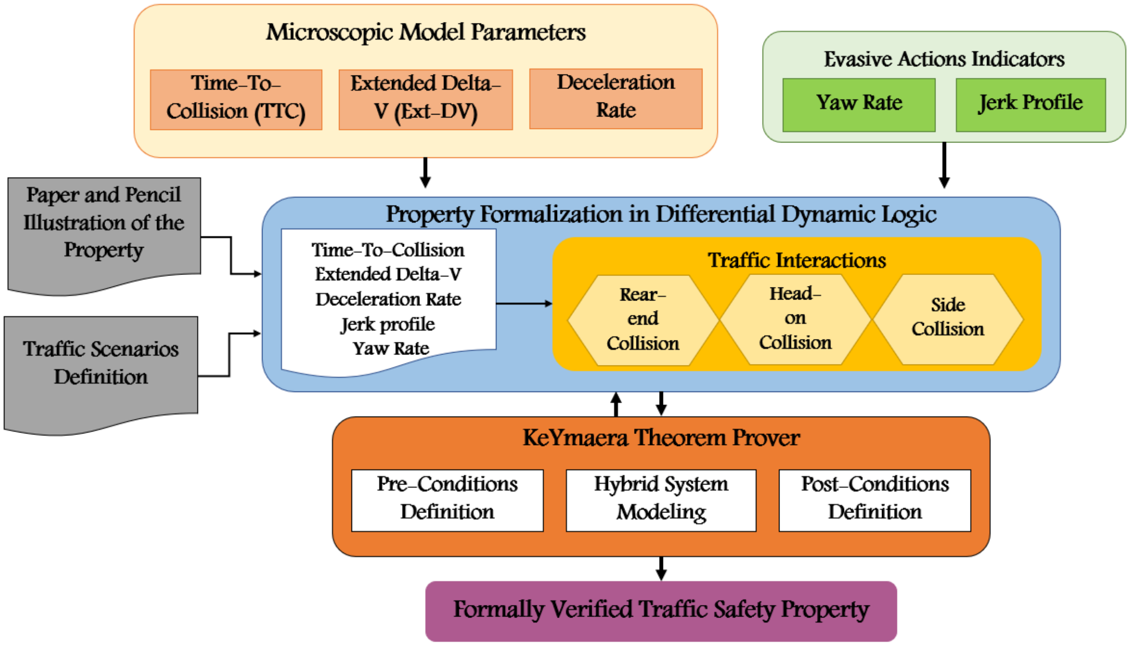

Figure 2.3: Methodology for Crash Severity Analysis

Table 2.2: Distribution of Deceleration-speed

Deceleration variable (m/s^2) **Speed interval (m/s)**	b	b1	b2	b3	B
[0, 20]	✓				
[20, 40]		✓			
[40, 60]			✓		
[60, 80]				✓	
> 80					✓

2.3 Formalization of TCT Safety Property

In this section, a description of the formed safety property is detailed along with its formalization in dL. Furthermore, the verification process of the formalized property in KeYmaera is described in different traffic scenarios.

2.3.1 Traffic Safety Property Definition

Using Extended Delta-V defined in Equation 2.8 and building on top of it, we end up defining a safety property. The latter combines different traffic conflict indicators, i.e., TTC, Extended Delta-V, jerk profile and yaw rate while covering multiple collision scenarios. This property aims to define the safe conditions under which the involved vehicles can mitigate traffic conflicts safely. The studied collision scenarios are right-side, head-on and rear-end collision, where two involved vehicles will potentially run into a collision induced by a traffic conflict situation. The latter will take place if one or all of the involved vehicles violate a traffic rule, show signs of bad driving or simply joins a zone where a conflict is already taking place. As mentioned earlier, a TTC value less than 3 seconds is considered critical for the involved vehicles since it restricts the driver to a short reaction time to take an evasive action after which the accident is imminent.

In this context, our reasoning is built on the TTC value and its defined threshold to start studying the safety of the involved vehicles. Once the computed value of TTC is less than 3 sec, the first tested evasive action is braking. The rate of deceleration is chosen based on the speed of the vehicle. Furthermore, the intensity of this rate is evaluated using the jerk profile to determine the sufficiency of the action. However, if the deceleration rate was not sufficient to avoid the collision then an added evasive action should be conducted to make sure that the accident will not take place. The complementing action, i.e., swerving is inspected using the defined yaw rate indicator and its effectiveness is also evaluated. In the case where TTC is less than 3 sec, braking is insufficient and swerving is unsuccessful, which may lead to a potential collision. The severity level of every collision is defined based on the speed of the vehicles during the conflict situation. We introduce the sketch of the defined safety property as follows:

$$\textit{(TCTs thresholds violated)} \longrightarrow \textit{[(Vehicles' Dynamics)*] (Collision)}$$

In order for an evasive action to be successful, the driver has to estimate its intensity based on certain parameters in real time. One of the main parameters to take into consideration in a traffic conflict is the current speed of the vehicle while entering the conflict. The speed of the involved vehicles is a key factor to determine the type of evasive action to execute and its intensity, since speed is crucial when it comes to the occurrence of accidents and their outcome. Based on the study done in [72] on different levels of speeds in rural and urban roads, a 1 mile/h (1.6 km/h) decrease of mean speed results on:

- around 6% of reduction in crash frequency for low average speeds on urban roads
- around 4% of reduction in crash frequency for medium speed on urban roads and low speed for rural roads
- around 3% of reduction in crash frequency for high speed on urban and main rural roads

Furthermore, Mclean et al. [45] demonstrated that every speed increase of 5 km/h taking place after reaching a speed of 60 km/h is more likely to double the possibility of a crash occurrence, thereby endangering the vehicles occupants. These stated arguments can only lead to one conclusion; in order for an evasive action to be successful, the reasoning must be built on the speed of the involved vehicles as detailed in Table 2.2. The aftermath of violating the defined safety conditions using traffic conflict indicators is studied in three different traffic scenarios and a formal analysis of the crashes severity is conducted depending on the situation. The defined severity level ranges from *property damage only*, *potential injury*, *non severe injury* to *severe injury*.

2.3.2 Formal Analysis of Crashes Severity in Different Traffic Scenarios

In this subsection, we describe the structure of the safety property combining TTC, SHW and SWV, followed by the definition of the formalization for every traffic scenario. Furthermore, we conduct a formal analysis of the crashes based on the type of collision and determine its severity level for every situation based on the speed of the involved vehicles when the collision occurs.

Formal Structure of the Property

Theorem 2.1 represents the structure of the property to be verified in KeYmaera. The components of the property are expanded and mathematically described by explicitly stating their formalization in the dL logic for every traffic scenario.

Theorem 2.1

$$\texttt{init} \wedge \texttt{TCT}_{violated} \longrightarrow \texttt{[(dyn)*](collision)}$$

Formalization and Verification Process in Traffic Conflict Scenarios

In order to conduct a crash severity analysis, we formalize the traffic safety property in three traffic scenarios, i.e., rear end, head on and side collisions, as highlighted by the methodology in Figure 2.3. Furthermore, we explicitly define the sketch of the formal property, i.e., *violated TCTs* and *collision*. As the for the pre-conditions definition and vehicles dynamics, i.e., *init* and *Vehicles' dynamics*, their mathematical definitions differ from one traffic scenario to the other. Therefore, their representation is defined for every scenario in the next section.

We cover the corner cases that a driver can face during a traffic conflict. With TTC less than 3 sec, the formalization of the described conditions for a collision to happen are listed below accompanied with their description.

$$\begin{aligned}
\texttt{TCT}_{violated} \equiv\ & (\texttt{TTC} \leq 3) \wedge (\Delta\texttt{V}_p < \Delta\texttt{V}) \wedge \\
& [((\texttt{J} \geq 0) \vee ((\texttt{r} \leq 0) \vee (\texttt{r} > 0.785))) \vee \\
& ((-9.9 < \texttt{J} < 0\ ) \wedge ((\texttt{r} \leq 0) \vee (\texttt{r} > 0.785))) \vee \\
& ((-15 < \texttt{J} \leq -9.9) \wedge ((\texttt{r} \leq 0) \vee (\texttt{r} > 0.785)))] \\
& \longrightarrow (\texttt{x}_1 = \texttt{x}_2)
\end{aligned}$$

1. **Delta-Vp < Delta-V:** The notation of ΔV is used instead of *Ext*-ΔV to simplify the use of the Extended Delta-V indicator. Moreover, ΔV_p indicates the computed speed variation **post** the deceleration. This condition stipulates that once the conflict is detected, the speed variation, i.e., ΔV_p, is less than the original value of Delta-V, i.e., ΔV, computed in a normal traffic flow. Under this situation, an imminent collision is bound to happen if no evasive action is taken, which leads us to investigate the following situations:

2. **(Jerk profile $\geq$ 0) & $\big($(Yaw rate $\leq$ 0) Or (Yaw rate $>$ 0.785)$\big)$:** These values represent the failure to execute evasive actions; no braking and no swerving which leads immediately to an accident, should the situation remain unchanged during the conflict situation.

3. **(-9.9 $<$ Jerk profile $<$ 0) & $\big($(Yaw rate $\leq$ 0) Or (Yaw rate $>$ 0.785)$\big)$:** In this case, the braking action reflects a deceleration rate demonstrating a normal braking situation which is not substantial enough to mitigate the conflict at hand safely. Furthermore, the yaw rate indicates that the trajectory was unchanged meaning no swerving was done. Once combined, these conditions eventually will lead to a collision that may or may not be severe depending on the involved vehicles' speed.

4. **(-15 $<$ Jerk profile $\leq$ -9.9) & $\big($(Yaw rate $\leq$ 0) Or (Yaw rate $>$ 0.785)$\big)$:** A jerk profile falling in the defined interval does not necessarily translate into a successful deceleration rate. In fact, the braking might not be enough to stop the vehicle before engaging in the collision. For this to be avoided, the deceleration can be accompanied with a minimum of swerving to make sure that the accident is avoided. The absence of the swerving in this case might be critical and can even lead to a collision that will have a certain impact depending on the vehicles' speeds.

The violation of the formalized safety property leads to an imminent collision that is formally expressed by the position of vehicle 1, i.e., x_1, equal to the position of vehicle 2, i.e., x_2.

$$\texttt{Collision} \equiv (\texttt{x}_1 = \texttt{x}_2)$$

For every traffic interaction, i.e., rear-end, head-on and right-side collision, the mathematical modelling of the pre-conditions and vehicles dynamics differ. Therefore, the formalization of *init* and *Vehicles' dynamics* is given below for every traffic interaction.

1. **Rear-end Collision:** For a rear-end collision, the speed of the following vehicle and its evasive actions in a traffic conflict say a lot about the severity of the crash should it occur. The assumed speed of vehicles 1 and 2 are denoted

by v_1 and v_2, respectively, where $v_2 > v_1$. The first step is to calculate the Extended Delta-V before a conflict and monitor the value of TTC, once the latter is equal or falls under 3 sec [26], an evasive action will be taken based on the value of v_2. After this action has taken place, the value of extended delta-V is calculated and compared to the initial value to determine if it is strong enough to avoid a potential crash. In case the TTC and Extended Delta-V thresholds are violated, an evasive actions-based analysis is conducted reporting the token actions to avoid the conflict and study them according to their computed intensity. The TTC formula for rear-end collisions is given in Equation 2.1 [46]. The formalization of the pre-conditions definition, i.e., *init*, given below, establishes different bounds for the defined parameters.

$$\begin{aligned}
\texttt{init} \equiv\ & (\texttt{v}_0 > \texttt{0}) \land (\texttt{v}_{post} \geq \texttt{0}) \land (\texttt{v}_1 \geq \texttt{0}) \land (\texttt{v}_2 \geq \texttt{0}) \land \\
& (\texttt{b} > \texttt{0}) \land (\texttt{b}_1 > \texttt{0}) \land (\texttt{b}_2 > \texttt{0}) \land (\texttt{b}_3 > \texttt{0}) \land \\
& (\texttt{B} > \texttt{0}) \land (\texttt{m}_1 > \texttt{0}) \land (\texttt{m}_2 > \texttt{0}) \land (\texttt{c} > \texttt{0}) \land \\
& (\texttt{L} > \texttt{0}) \land (\texttt{rd} > \texttt{0}) \land (\texttt{v}_2 > \texttt{v}_1) \land (\texttt{x}_2 < \texttt{x}_1) \land \\
& (\texttt{b}_1 > \texttt{b}) \land (\texttt{b}_2 > \texttt{b}_1) \land (\texttt{b}_3 > \texttt{b}_2) \land (\texttt{B} > \texttt{b}_3) \land \\
& (\texttt{TTC = } \frac{x_1 - x_2 - L}{v_2 - v_1}) \land (\theta_2 = \frac{x_2}{rd})
\end{aligned}$$

The dynamics of the vehicle are modeled using its position x_2, velocity v_2 and acceleration a_2. The formalization of the ordinary differential equation linking these parameters in KeYmaera is given along with the derivation of the jerk profile and yaw rate.

$$\begin{aligned}
\texttt{dyn} \equiv\ & \{(\texttt{x}_2\texttt{'}) = (\texttt{v}_2),\ (\texttt{v}_2\texttt{'}) = (\texttt{a}_2),\ (\texttt{a}_2\texttt{'}) = (\texttt{J}_2), \\
& (\theta_2\texttt{'}) = (\texttt{r}_2),\ (\texttt{t'}) = (\texttt{1})\}
\end{aligned}$$

A proof sketch of this property is provided in Figure 2.4 where the violated threshold of the TTC indicator, defined at the top, represents the first flag for an upcoming conflict. Subsequently, the formal analysis of crashes is carried out for every specific speed interval and based on the initial deceleration rate. Therefore, each branch represents a sub-goal, which is automatically proved by KeYmaera, thereby proving the property.

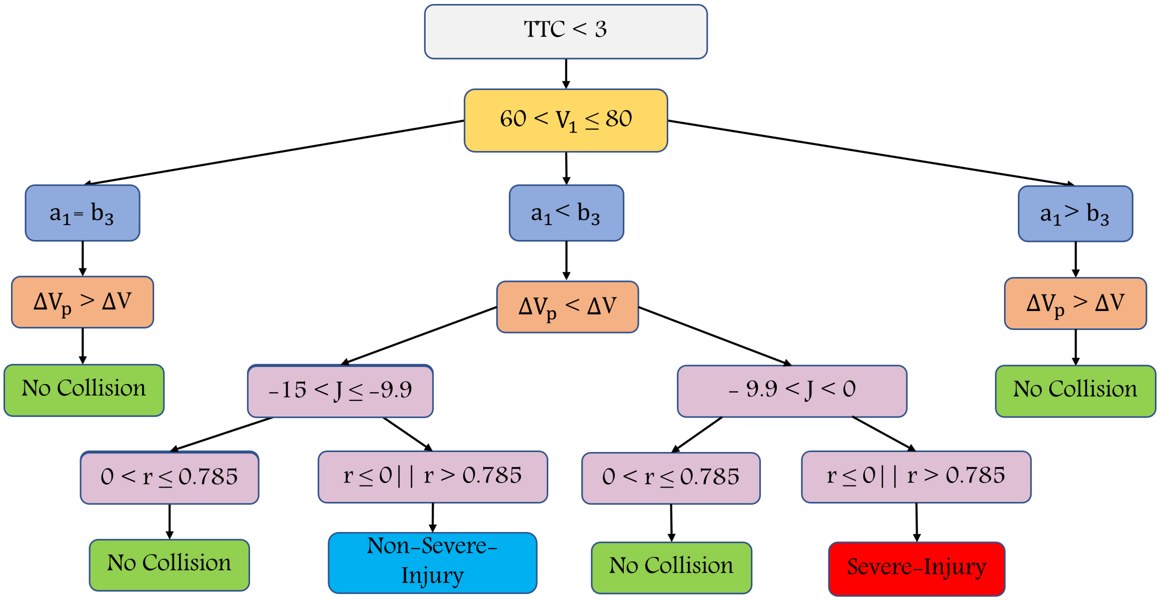

Figure 2.4: Proof Structure of the Property in a Rear-end Collision Scenario

For a rear-end scenario, we focus on the following vehicle and study its behavior according to the traffic variations caused by the leading vehicle, e.g., braking and swerving. The proof structure depicted by Figure 2.4 is mainly used to explain the process, we only provide one speed interval, i.e., [60,80] km/h for simplicity.

2. **Head-on Collision:** For a head-on collision, the velocities are considered as scalars and both of the vehicles' speeds vary and fall into different intervals where the TTC is calculated. Once a traffic conflict situation is identified (TTC < 3), the magnitude of the evasive action is carried out based on the speed interval. This magnitude is analysed in order to conclude if the crash will happen or not and determine its severity in both cases. Based on the computed value of Extended Delta-V, a series of actions, i.e., braking and swerving, are executed in order to avoid the conflict safely. However, the execution of evasive actions in some cases proves either insufficient to avoid a collision or too strong for the situation at hand leading to further complications. For this, we build our study on analysing the intensity of the taken actions by analysing the obtained values for the rates indicators, jerk profile and yaw rate, as measures for braking and swerving intensities, respectively. The formalization of the pre-conditions definition for this traffic scenario are described below.

$$
\begin{aligned}
\texttt{init} \equiv\ & (\texttt{v}_0 > 0) \land (\texttt{v}_{post} \geq 0) \land (\texttt{v}_1 \geq 0) \land (\texttt{v}_2 \geq 0) \quad \land \\
& (\texttt{b} > 0) \land (\texttt{b}_1 > 0) \land (\texttt{b}_2 > 0) \land (\texttt{b}_3 > 0) \quad \land \\
& (\texttt{B} > 0) \land (\texttt{m}_1 > 0) \land (\texttt{m}_2 > 0) \land (\texttt{c} > 0) \quad \land \\
& (\texttt{rd} > 0) \land (\texttt{x}_2 \neq \texttt{x}_1) \land (\texttt{TTC} = \frac{x_1 - x_2}{v_1 + v_2}) \quad \land \\
& (\texttt{b}_1 > \texttt{b}) \land (\texttt{b}_2 > \texttt{b}_1) \land (\texttt{b}_3 > \texttt{b}_2) \land (\texttt{B} > \texttt{b}_3) \quad \land \\
& (\theta_1 = \frac{x_1}{rd}) \land (\theta_2 = \frac{x_2}{rd})
\end{aligned}
$$

As for modelling the system dynamics, the corresponding ODE is given for the involved vehicles, i.e., vehicle 1 and vehicle 2, using their positions x_1 and x_2, velocities v_1 and v_2, and accelerations a_1 and a_2. Furthermore, the jerk profile and yaw rate are computed for both vehicles and denoted as J_2, J_2, r_1 and r_2, respectively. The formalization of the systems dynamics in dL is given below:

$$
\begin{aligned}
\texttt{dyn} \equiv \{ & (\texttt{x}_1\texttt{'}) = (\texttt{v}_1),\ (\texttt{v}_1\texttt{'}) = (\texttt{a}_1),\ (\texttt{a}_1\texttt{'}) = (\texttt{J}_1), \\
& (\theta_1\texttt{'}) = (\texttt{r}_1), \\
& \quad (\texttt{x}_2\texttt{'}) = (\texttt{v}_2),\ (\texttt{v}_2\texttt{'}) = (\texttt{a}_2),\ (\texttt{a}_2\texttt{'}) = (\texttt{J}_2), \\
& \quad (\theta_2\texttt{'}) = (\texttt{r}_2),\ (\texttt{t'}) = (1)\}
\end{aligned}
$$

A proof sketch of this property is provided in Figure 2.5 where the violated threshold of the TTC indicator, defined at the top, represents the first flag for an upcoming conflict. Subsequently, the formal analysis of crashes is carried out for every specific speed interval and based on the initial executed deceleration. Therefore, multiple branches are evaluated, where each branch leads to a sub-goal that is automatically proved by KeYmaera. Following the branches and proving every possibility leads to the verification of all sub-goals, and hence proving the property. This proof structure is mainly used to explain the process, by providing two speed intervals, i.e., [60,80] km/h and [20,40] km/h for vehicle 1 and 2, respectively. For simplicity, we study their actions in a traffic conflict and identify the critical situations and the requirements in order to mitigate them safely.

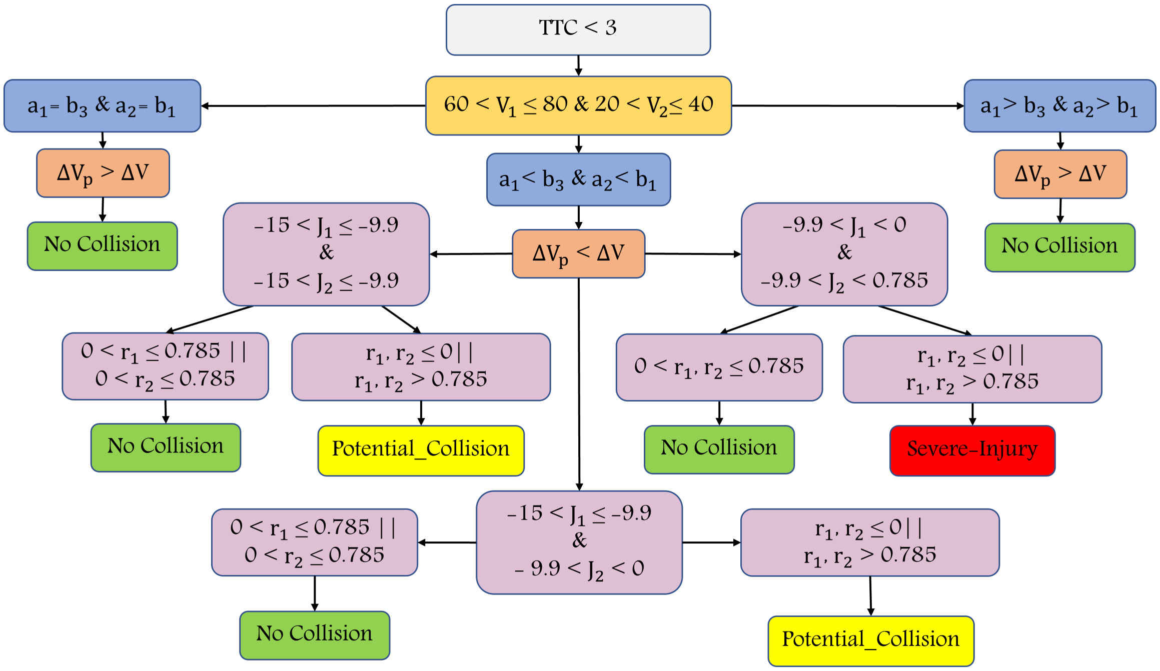

Figure 2.5: Proof Structure of the Property in a Head-on Collision Scenario

3. **Side Collision:** A conflict situation taking place from the driver's right side on the main street is prone to happen under many favorable conditions. These situations such as a turning vehicle that does not decelerate nor stop to make sure that the main road is empty, or the main street driver reacting late to the threat coming from the side. In this scenario, the angle of the collision varies depending on the turning vehicle's trajectory. For this, the defined Delta-V formula explicitly defines the angle using its *cosine* value. Using the formula defined in [33], the Extended Delta-V for vehicle 1 is calculated while taking into consideration the collision angle α as shown in Equation 2.15.

$$\Delta V_1 = \frac{m_2}{m_1 + m_2} . \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos\alpha} \tag{2.15}$$

where m_1, m_2, v_1, v_2 are the masses and speeds of the vehicles 1 and 2, respectively, and α is the approaching angle. In this traffic scenario, TTC is defined based on the positioning of the vehicles as described by Equations 2.5 and 2.6. The formalization of the of the defined parameters as pre-conditions, *init*, is given below.

$$
\begin{aligned}
\mathtt{init} \equiv\ & (\mathtt{v}_0 > \mathtt{0}) \land (\mathtt{v}_{post} \geq \mathtt{0}) \land (\mathtt{v}_1 \geq \mathtt{0}) \land (\mathtt{v}_2 \geq \mathtt{0}) \land \\
& (\mathtt{b} > \mathtt{0}) \land (\mathtt{b}_1 > \mathtt{0}) \land (\mathtt{b}_2 > \mathtt{0}) \land (\mathtt{b}_3 > \mathtt{0}) \land \\
& (\mathtt{B} > \mathtt{0}) \land (\mathtt{m}_1 > \mathtt{0}) \land (\mathtt{m}_2 > \mathtt{0}) \land (\mathtt{c} \geq \mathtt{-1}) \land \\
& (\mathtt{b}_1 > \mathtt{b}) \land (\mathtt{b}_2 > \mathtt{b}_1) \land (\mathtt{b}_3 > \mathtt{b}_2) \land (\mathtt{B} > \mathtt{b}_3) \land \\
& (\mathtt{t}_{f1} > \mathtt{0}) \land (\mathtt{t}_{f2} > \mathtt{0}) \land (\mathtt{t}_{r1} > \mathtt{0}) \land (\mathtt{t}_{r2} > \mathtt{0}) \land \\
& (\mathtt{t}_{r1} > \mathtt{t}_{f1}) \land (\mathtt{t}_{r2} > \mathtt{t}_{f2}) \land (\theta_1 = \frac{x_1}{rd}) \land (\mathtt{c} \leq \mathtt{1}) \land \\
& (\theta_2 = \frac{x_2}{rd}) \land (\mathtt{rd} > \mathtt{0}) \land (\mathtt{x}_2 \neq \mathtt{x}_1)
\end{aligned}
$$

The formalization of the ODE modeling the system dynamics is given below for the involved vehicles, i.e., vehicle 1 and vehicle 2, using their positions x_1 and x_2, velocities v_1 and v_2, and accelerations a_1 and a_2. Moreover, the jerk profile and yaw rate are computed for both vehicles and denoted as J_1, J_2, r_1 and r_2, respectively.

$$
\begin{aligned}
\mathtt{dyn} \equiv\ & \{(\mathtt{x}_1\mathtt{'}) = (\mathtt{v}_1),\ (\mathtt{v}_1\mathtt{'}) = (\mathtt{a}_1),\ (\mathtt{a}_1\mathtt{'}) = (\mathtt{J}_1), \\
& (\theta_1\mathtt{'}) = (\mathtt{r}_1), \\
& \quad (\mathtt{x}_2\mathtt{'}) = (\mathtt{v}_2),\ (\mathtt{v}_2\mathtt{'}) = (\mathtt{a}_2),\ (\mathtt{a}_2\mathtt{'}) = (\mathtt{J}_2), \\
& \quad\ (\theta_2\mathtt{'}) = (\mathtt{r}_2),\ (\mathtt{t'}) = (\mathtt{1})\}
\end{aligned}
$$

4. **Right-side Collision:** For a right-side collision, the angle of collision α can vary depending on the turning angle of the vehicle coming from the right, knowing that the other vehicle is following a straight line. For this case, $\alpha = 90°$ is used to simplify things with a conflict-to-collision region S defined as shown in Figure 2.1, where the two involved vehicles intercept each other if the crash is not avoided. A proof sketch of this property is provided in Figure 2.6 where the violated threshold of the TTC indicator, defined at the top, represents the first flag for an upcoming conflict. Subsequently, the formal analysis of crashes is carried out for every specific speed interval, based on the initial deceleration value. Therefore, multiple branches are evaluated, where each branch leads to a sub-goal that is automatically proved by KeYmaera. Following the branches and verifying every possibility leads to the verification of all sub-goals, thereby proving the correctness of the property. For simplicity, this proof structure is

mainly to explain the process. Thereby, we only provide two speed intervals, i.e., [40,60] km/h and [20,40] km/h for vehicles 1 and 2, respectively, in order to study their actions in a traffic conflict and identify the possible critical situations and the requirements to safely mitigate them.

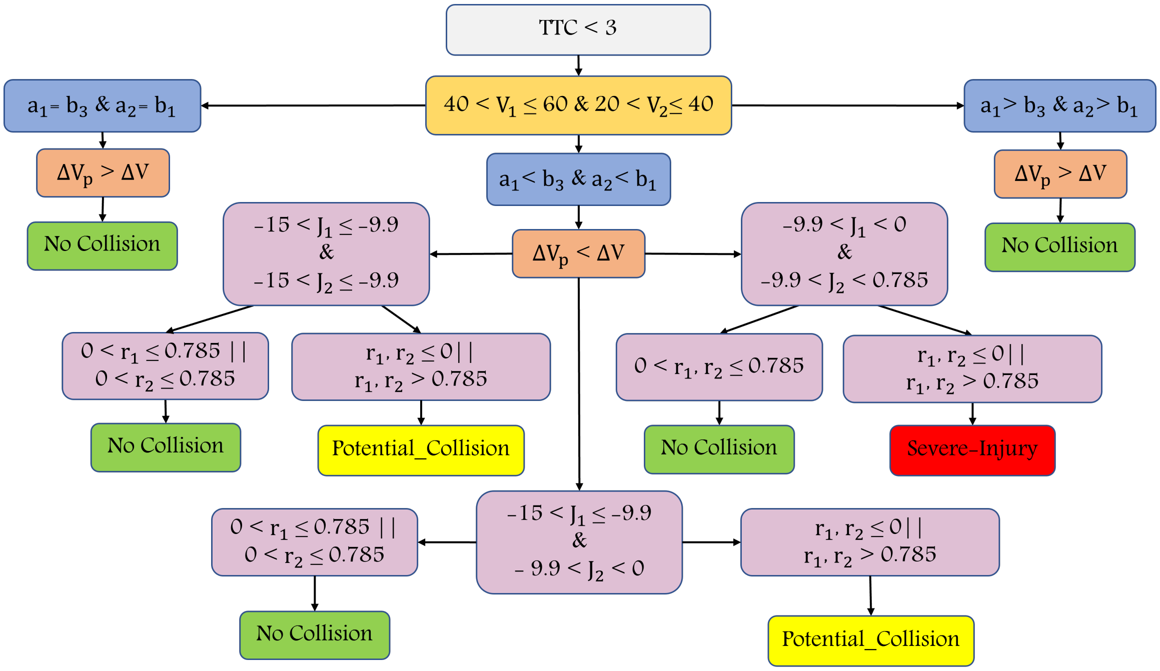

Figure 2.6: Proof Structure of the Property in a Right-side Collision Scenario

2.4 Experimental Results

Based on the proposed methodology in Figure 2.3, we defined, in the previous sections, the traffic safety property specifying the crashes severity level, followed by its formalization using differential dynamic logic with the hybrid system dynamics modeled. We now discuss the proving process in KeYmaera, where a series of automatic proof strategies are applied. For instance, the property of the hybrid program including a differential equation requires solving the dynamics of hybrid systems, i.e., vehicles. Afterwards, the obtained solutions are used to prove each sub-goal of the property until reaching the main goal. Eventually, KeYmaera's verification process results in either a proved property or an error which disables the prover from continuing the process. In our case, the correctness of the property is proved for the three different scenarios of rear-end, head-on and right-side collision. Table 2.3 describes the number of rules applied to simplify the property and prove it along with the number of proved sub-goals and the time duration of the proof in KeYmaera running on a computer with Intel(R) Core(TM) i7-1065G7 CPU @ 1.30GHz and 16GB of RAM.

We managed to identify the performance indices based on their impact in the proved property and obtained results. In fact, having different parameters taken into account can only make the process and results more accurate and trustworthy. Studying the impact of certain indicators used in the formalization, we are able to cover different scenarios and extract different results based on the conditions and factors put in place for every case. Our performance measures are Time-To-Collision indicator in traffic conflict, initial speed of vehicles, Extended Delta-V, the intensity of the evasive actions reflected by the Jerk Profile and Yaw Rate, respectively. These measures are the key to prove if the property holds or not.

Table 2.3: KeYmaera Verification Results for Different Collision Scenarios

Scenario of Collision	**Number of Applied Rules**	**Number of Proved Goals**	**CPU Verification Time (sec)**
Rear-end	1438	239	11
Head-on	1859	373	27.7
Right-side	1284	209	10.4

2.5 Summary

In this chapter, a transportation safety property was defined and formalized using the hybrid theorem prover, KeYmaera. As depicted by the methodology Figure 2.3, the defined property puts to use certain TCTs, i.e., TTC and Extended Delta-V, and evasive actions indicators, i.e., in order to formally analyse crashes severity based on their computed values. The outcome of the property is either a collision between involved vehicles or no collision by safely mitigating the situation. The analysis of the taken actions during the conflict determines the severity of the outcome, based on which, the crash severity level is deduced. However, knowing that traffic conflicts differ from one situation to another, we propose to conduct our analysis in three different traffic scenarios, i.e., rear-end, head-on and right-side collision. The description of every scenario and its specifications along with the severity level of the crash is also provided. Furthermore, the intensity of braking and/or swerving is evaluated according to the pre-defined thresholds for the employed evasive actions indicators. Based on this evaluation and taking into consideration multiple parameters, e.g., the type of conflict, vehicles' speeds and initial deceleration rates, we deduce the outcome of the traffic conflict. Furthermore, we propose different speed intervals and for each interval, a deceleration rate is associated in order to cover as many cases as possible.

The work developed in this chapter demonstrates the importance of combining different traffic conflicts indicators as a unified safety property, where each indicator provides a complementary safety aspect of the interactions. In practice, an application for this formal analysis will lead to a reduction in the number of crashes in specific situations, and it will impact the transportation field by increasing the overall safety level. Towards this goal, in the next chapter we propose a formal analysis of a common traffic conflict considered as the reason behind an important number of crashes in car following models, namely, shockwaves, by investigation its association with two conventional TCTs, i.e., Time-To-Collision and Space Headway.

Chapter 3

Formalization of the TTC, Space Headway and Shockwave Property

This chapter covers in detail the definition of a safety transportation property combining Time-To-Collision (TTC), Space Headway (SHW) and Shockwave (SWV) which are the main Surrogate Safety Measures (SSM) used in this thesis. We start by formally verifying the traffic safety property using KeYmaera, then, we use a traffic simulation tool called SUMO in order to validate the formalized property. The traffic simulation outcome validates the formalized property by reflecting its correlation with traffic conflicts as will be detailed in the rest of the chapter.

3.1 Methodology for TTC-SWV Bidirectional Relation

In car following models [20], rear-end crashes occur frequently due to different traffic events. The study in [8] identified shockwaves as a traffic conflict leading to the occurrence of rear-end crashes. Shockwaves are traffic events that occur due to predicted and unpredicted changes in the traffic state, such as crashes and intersections. In order to further study the impact of shockwaves on the traffic flow, we analyse the variation of two significant TCTs, i.e., Time-To-Collision (TTC), Space Headway (SHW), known to reflect minor disruptions in traffic flow. This investigation of the link between TTC, SHW and Shockwave (SWV), which is defined as the indicator for the occurrence of shockwaves, is applied to improve the safety of traffic by predicting future traffic conflicts and taking the right measures to avoid them.

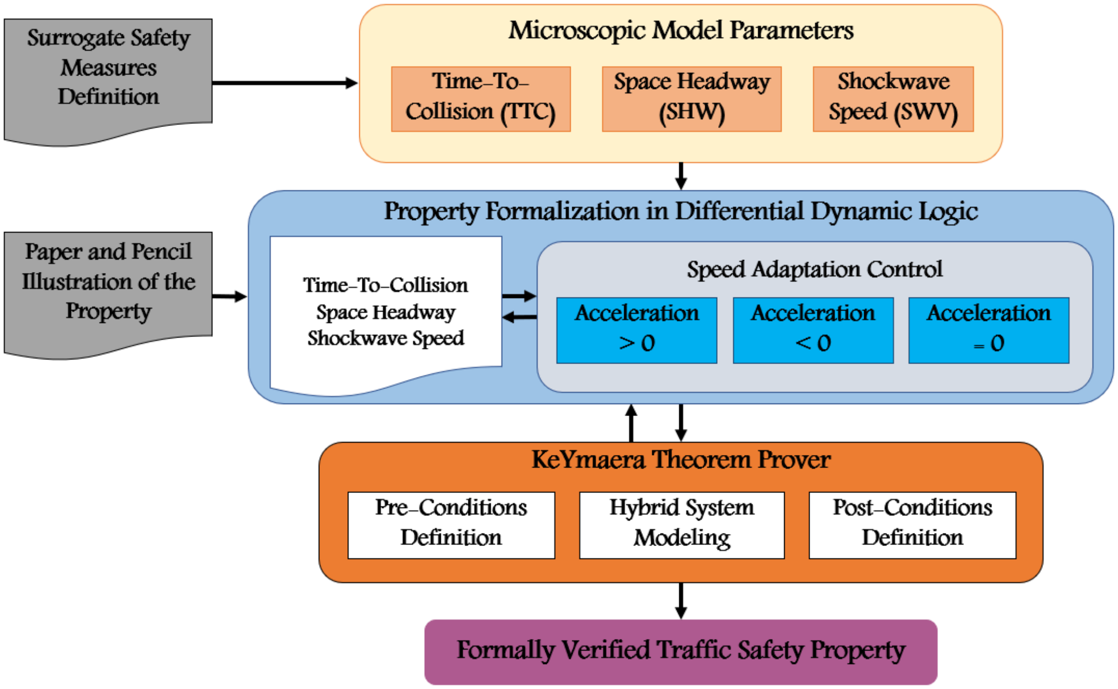

Figure 3.1: Methodology for TTC-SWV Bidirectional Relation

As depicted by Figure 3.1, we propose a new traffic safety property combining SWV, TTC and SHW. We start by providing a mathematical definition of the applied TCTs in this property. In order to construct a formal model of the proposed safety traffic property, we provide a formalization of the introduced SSMs using differential dynamic logic (dL). Subsequently, we introduce a paper and pencil illustration of property resulting in a bidirectional relation between the mentioned indicators, where we study the impact of violating the TTC and SHW thresholds on the occurrence of shockwaves, by determining the shockwave speed, and vice versa. Furthermore, we integrate a speed adaptation mechanism in order to update the speed of new vehicles joining the queue. This speed adaptation process consists of assigning the suitable acceleration values to safely mitigate the traffic conflict and avoid future conflicts. Based on the assigned acceleration, the speed is either increased, decreased or maintained the same. Consequently, we formalize the traffic safety property along with the speed adaptation process in dL. Using KeYmaera, we verify the formalized property. This adaptation mechanism proves its efficiency by reducing upcoming traffic conflicts due to the early notice informing the vehicles about the traffic state ahead. Moreover, the formalized property is introduced to the hybrid theorem prover tool, KeYmaera, to be formally verified.

3.2 Formalization of the TTC, SHW and SWV Safety Property

In order to define the traffic safety property and formalize it, it is essential to define the used TCTs. Furthermore, a detailed description of the property is provided in the rest of the section highlighting the link between TTC, SHW and SWV.

3.2.1 Mathematical Modeling of the TCTs

Time-To-Collision

In this chapter, we propose a traffic safety property linking TTC, SHW and SWV for car following models. The latter can be represented by platoons of vehicles where every vehicle can be a leader and/or a following vehicle. In order to mathematically model the TTC indicator, the number of vehicles is generic which explains the generalization of Equation 2.1 resulting in Equation 3.1 [46].

$$TTC = \frac{x_i - x_{i+1} - L_i}{v_{i+1} - v_i}, \qquad v_{i+1} > v_i \tag{3.1}$$

where vehicle i and vehicle *i+1* are the leading and following vehicles, respectively, x_i, x_{i+1}, v_i and v_{i+1} are the positions and velocities of vehicles i and *i+1*, respectively, and L is the length of vehicle i.

Space Headway

The Space Headway indicator (SHW) describes the physical distance separating the front bumps of every two consecutive vehicles. The space headway is the position difference between vehicle i and *i+1* can be mathematically defined in Equation 3.2 as follows:

$$SHW = x_i - x_{i+1} \tag{3.2}$$

where x_i and x_{i+1} are the positions of vehicle i, the leading vehicle, and the following vehicle *i+1*, respectively.

Shockwave Speed

A shockwave is a macroscopic event that occurs during a traffic flow as a result of different factors such as signalized intersection, aggressive lane change causing the following vehicles to brake sharply, and a collision downstream the platoon of vehicles. A shockwave can be identified by a platooning of stationary vehicles or slowed vehicles on a certain road segment. Mathematically, the occurrence of shockwaves can be detected by computing the shockwave speed (SWV) defined over a range of consecutive vehicles, the formula in the macroscopic model is given in Equation 3.3 [67].

$$SWV = \frac{q_i - q_j}{k_i - k_j} \tag{3.3}$$

where q_i, q_j, k_i and k_j represent the traffic flow and flow density for traffic state i representing the congested state, and for traffic state j representing the uncongested state, respectively. However, due to the scarcity of the traffic data or its delay, we opted for analysing shockwave events at the microscopic level. Furthermore, conducting the traffic analysis at a microscopic level has its advantages when it comes to capturing the vehicles' dynamics as well as the drivers' actions in order to conduct a safety analysis. Therefore, the shockwave speed is now defined at the microscopic level using the dynamics of a range of consecutive vehicles on a road segment. Based on the work done in [67], the flow density (k) is found to be equal to the inverse of the headway distance (SHW) at the microscopic level as shown below in Equation 3.4, where the space headway (SHW) is defined as the physical distance separating two consecutive vehicles, for example, the second and third vehicle in row.

$$k = \frac{1}{SHW} \tag{3.4}$$

Furthermore, using the conventional definition of traffic flow in engineering, the traffic flow (q) is reduced to the vehicle speed multiplied by the flow density (k) as shown in Equation 3.5. However, by substituting (3.4) into (3.5) yields Equation 3.6 as the microscopic expression for traffic flow [67].

$$q = v * k \tag{3.5}$$

$$q = \frac{v}{SHW} \tag{3.6}$$

Redefining SWV at the microscopic level as the shockwave speed of a platoon of vehicles in a car following model, we replace (3.4) and (3.6) into (3.3) to yield Equation 3.7 [67].

$$SWV = \frac{\dfrac{v_i}{SHW_i} - \dfrac{v_j}{SHW_j}}{\dfrac{1}{SHW_i} - \dfrac{1}{SHW_j}} \tag{3.7}$$

where v_i and v_j represent the speed of vehicle i and j, respectively. As for SHW_i and SHW_j, they represent the distance separating vehicles i and $i+1$, and vehicles j and $j+1$, respectively, with $i \neq j$.

3.2.2 Bidirectional Causal Relation Between TTC, SHW and SWV

In this part, we introduce a bidirectional relation, where the reduction of TCT computed values, i.e., TTC and SHW can produce a shockwave. On the other hand, the existence of shockwaves is capable of impacting the two indicators values once present. Using the equations defined earlier, i.e., Equations 3.1, 3.2 and 3.3 for TTC, SHW and SWV, respectively, we start by defining the property in dL in order to verify it using KeYmaera. The formalization sketch of the property, given by:

$$\textit{Violated Indicators } (Ind_{violated}) \longleftrightarrow \textit{Shockwave Occurrence } (SWV_{speed})$$

is formally expressed in Theorem 3.1, where $ind_{violated}$ represents a set of defined pre-conditions on system variables and $SWV_{violated}$ defines the shockwave speed bounds.

Theorem 3.1

$$\texttt{init} \longrightarrow \texttt{[(dyn)*]}\big(\texttt{Ind}_{violated} \longleftrightarrow \texttt{SWV}_{speed} < 0 \vee 0 \leq \texttt{SWV}_{speed} \leq 7\big)$$

The formalization of the pre-conditions, i.e., *init*, in dL, is given by:

```
init ≡
 ∀ C i.
   ∀ C j.  ((i ≠ j) ∧ (v(i) > 0) ∧ (v(j) > 0) ∧
      (x(i) < x(j)) ∧ (v(i) > v(j))
 ∧ (∀ C k.  ((k ≠ j) ∧ (k ≠ i) ⟶ (x(j) < x(k)) ∧
             (v(k) > 0))
     ∧ (SHW = x(j) - x(i)) ∧ (d(i) = x(j) - x(i)) ∧
             (d(k) = x(k) - x(j))
     ∧ (SWV_speed = ($\frac{v(k)}{d(k)}$ - $\frac{v(i)}{d(i)}$)/($\frac{1}{d(k)}$ - $\frac{1}{d(i)}$)) ∧
             (TTC = $\frac{SHW - L}{v(i) - v(j)}$))
 ∧ (K = $\frac{N}{1000}$) ∧ (A > 0) ∧ (C > 0) ∧ (L > 0))
```

where the universal quantifier ∀ C reflects that the formalization is carried out for all objects of sort C, C being a built-in sort in KeYmaera used here to represent cars [58]. The employed indicators are formalized in dL, along with defining the bounds of the used variables, e.g., vehicles' positions and speeds.

As for the dynamics of the vehicles, we model their positions x_i, velocities v_i and accelerations a_i in dL. The formalization of the ODE linking these parameters in KeYmaera is given as:

```
dyn ≡ {∀ C i.  (x(i)') = (v(i)),
         ∀ C i.  (v(i)') = (a(i)), (t') = (1)}
```

where the derivative x_i' of x_i and v_i' of v_i over time are $\frac{\mathrm{d}x_i}{\mathrm{d}t}$ and $\frac{\mathrm{d}v_i}{\mathrm{d}t}$, respectively.

We now define the thresholds of the temporal proximity indicator, i.e., TTC, along with the spatial proximity indicator, i.e., SHW, in dL as given below, in order to set the safety constraints for vehicles driving in a zone where a shockwave is detected.

$$\begin{aligned}
\texttt{Ind}_{violated} \quad \equiv \quad & \forall\ \texttt{C i.}\\
& \forall\ \texttt{C j.}\\
& \quad ((\texttt{i} \neq \texttt{j})\ \wedge\\
& \qquad \texttt{TTC} < \texttt{3} \wedge \texttt{SHW} < \frac{1}{k})
\end{aligned}$$

The shockwave speed threshold is formally defined for all cars as:

$$\begin{aligned}
\texttt{SWV}_{speed} \equiv\ & \forall\ \texttt{C i.}\\
& \forall\ \texttt{C j.}\\
& \quad ((\texttt{i} \neq \texttt{j})\ \wedge\\
& \qquad \texttt{0} \leq \texttt{SWV} \leq \texttt{7} \vee \texttt{SWV} < \texttt{0})
\end{aligned}$$

Time-To-Collision and Space Headway Causing Shockwaves

As defined in Equation 3.1, TTC is used to determine if a situation is critical or not based on its computed value. A TTC value in a range of 0 to 3 seconds indicates an endangering traffic situation that needs the immediate attention of the involved car drivers. To satisfy the same objective, TTC combined with the SHW indicator can open new portals to analyse an existing traffic situation. Assuming the existence of a platooning of vehicles on a road section, the density k is defined as the average number of vehicles that occupy one mile or one kilometer of road space and expressed in vehicles per mile or per kilometer. The mathematical modeling of the density is given by Equation 3.8, where N is the number of vehicles using the road section.

$$k = \frac{N}{1000} \tag{3.8}$$

In a conflict-free traffic flow, SHW is calculated as the inverse of the density of a certain road section as defined by Equation 3.9. For a safe spacing between vehicles, SHW value should be greater than or at least equal to $(\frac{1}{k})$.

$$SHW \geq \frac{1}{k} \tag{3.9}$$

A TTC less than 3 sec accompanied with a SHW less than its threshold between multiple consecutive vehicles will have noticeable implications on the traffic flow. The main consequence of these conditions is the formation of congested areas where the flow diminishes and vehicles slow down and add up to form a queue characterizing a shockwave. The existence of a shockwave can be detected by determining its speed. Based on the work done by Ibrahim et al. in [29], a shockwave speed of 7m/s, i.e., 25.2km/h calculated between the ith and jth vehicles in a platoon of vehicles, where $i \neq j$ provokes a shockwave. This detected shockwave can propagate either upstream or downstream depending on the sign of the speed value. For a speed value:

- SWV < 0 : the shockwave propagates in the same direction as the traffic stream, i.e., upstream
- SWV ≥ 0 & SWV < 7 : the shockwave propagates against the traffic stream,i.e., downstream

The property is used to prove that the presence of a shockwave can be induced by a TTC that is less than 3 and a noticeable reduction of the space headway over a platoon of vehicles. Nevertheless, these observations are made over a range of vehicles where speed variation between vehicles differ from one driver to the other according to the traffic environment at hand (signalized intersection, accident occurrence ahead, etc.). Consequently, we deduce an implication relationship describing the defined preconditions and their consequence over a platoon of vehicles. The formalization of the implication of the right-hand side (RHS) is given by:

$$
\begin{array}{l}
\texttt{Ind}_{violated} \longrightarrow \texttt{SWV}_{speed} \equiv \\
\quad \forall\ \texttt{C i.} \\
\quad\ \ \forall\ \texttt{C j.} \\
\qquad ((\texttt{i} \neq \texttt{j}) \wedge \texttt{TTC} < 3 \wedge \texttt{SHW} < \dfrac{1}{k} \longrightarrow 0 \leq \texttt{SWV} \leq 7 \vee \texttt{SWV} < 0)
\end{array}
$$

Shockwaves Causing Time-To-Collision and Space Headway

Inspired by this line of thought, we decided to explore the other direction of the implication where the presence of the shockwave might cause changes in the traffic flow by reducing the spacing between consecutive vehicles leading to a reduced time-to-collision over a platoon of vehicles. The impact of the shockwave is noticed by focusing on the vehicles forming the queue, however, its propagation is observed by monitoring the vehicles joining the platoon recently and analysing their related TTC and SHW. As a demonstration example, Figure 3.2 presents the propagation of the shockwave against the traffic stream, i.e., downstream. In general, the direction of propagation can be determined based on the sign of the shockwave speed, for example, a negative sign confirms a downstream propagation. In this case, the vehicles entered a signalized intersection where the long light duration causes the formation of a queue of stand by vehicles. Region A is a congested area while region B is an uncongested area, where the traffic flow runs smoothly and uninterrupted. As a result, the traffic flow in the congested area is lower than the traffic flow in the uncongested region, i.e., $q_A < q_B$. Furthermore, this traffic event will impact the mean speed by causing a reduction of the mean speed in the congested area, i.e., $V_A < V_B$ in addition to an increase of traffic density in state A compared to state B, i.e., $k_A > k_B$. For the vehicles joining with high speed, their braking will be abrupt and strong in order to stop the vehicle without crashing into the vehicle in the front which will leave a small spacing between the two vehicles in addition to a smaller time to collision. The computed values of the two indicators, in this condition of a shockwave, classify the situation as a traffic conflict where certain measures should be taken to mitigate it safely. The formalization of the left-hand side (LHS) implication for all objects of sort C can be described as:

```
$\texttt{SWV}_{speed} \longrightarrow \texttt{Ind}_{violated} \equiv$
  $\forall$ C i.
    $\forall$ C j.
      $0 \leq \texttt{SWV} \leq 7 \vee \texttt{SWV} < 0 \longrightarrow \texttt{TTC} < 3 \wedge \texttt{SHW} < \frac{1}{k}$
```

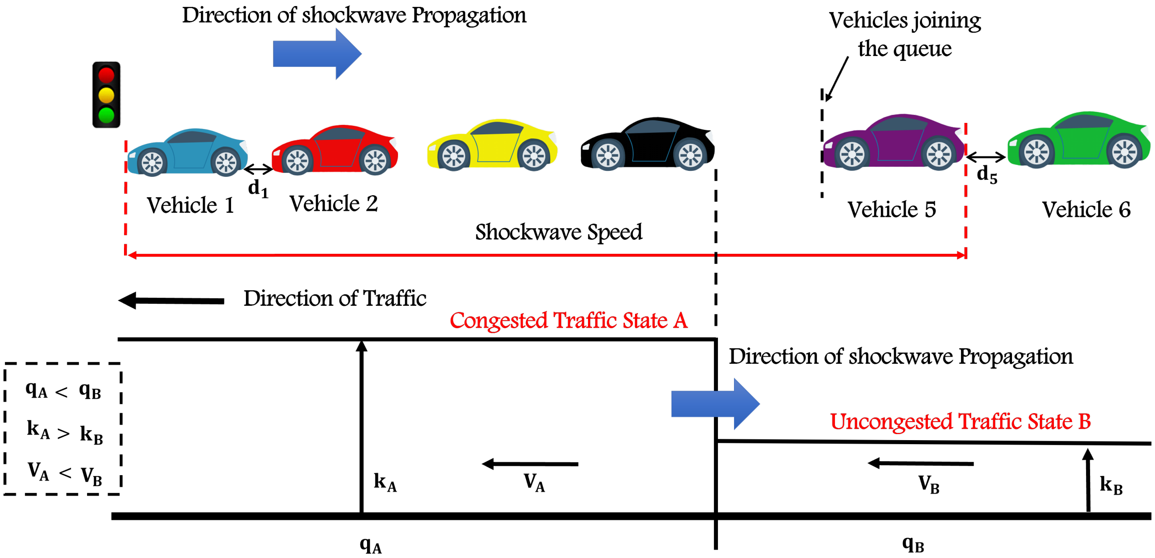

Figure 3.2: Shockwave Downstream Propagation

3.3 Validation and Experimental Results

In this section, we provide an overview of the proposed traffic simulation tool, along with a description of the real-life traffic dataset. The process of the property validation is detailed in the rest of the section.

3.3.1 SUMO: A Traffic Systems Simulation Tool

In the microscopic model, the whole focus is directed to the particulate individual vehicles during the traffic flow of a platoon of vehicles. Micro-simulation [52] is introduced to retrieve the motion parameters of the vehicle in question as well as analysing its interaction with adjacent vehicles. This analysis aims to evaluate the effects of proposed safety solutions. Furthermore, micro-simulation represents predicted traffic behavior as well as modeling congested road networks thanks to its ability to simulate queuing conditions. Moreover, it is capable of simulating the behaviour of individual vehicles within a predefined road network and is used to predict the likely impact of changes in traffic patterns resulting from changes to traffic flow or to the physical environment.

In order to perform a thorough analysis of the traffic flow, we employ a traffic simulation tool, namely Simulation Urban MObility (SUMO) [37], which will allow us to conduct a trajectory analysis of the vehicle along with an extraction of the needed parameters. As a micro-simulation tool, SUMO is an open-source traffic simulator that provides different tools and packages for every step of traffic network simulation [52]. SUMO comes with a ready to use and adjustable car-following model that allows for a flexible calibration and validation process. It also provides SSM devices which can be attached to vehicles to record conflicts and interactions between vehicles and calculate surrogate safety measures. The extracted traffic safety measures are time-to-collision (TTC), Space Headway (SHW), vehicles' velocities, acceleration and headway among other indicators.

3.3.2 Description of the Real-life Dataset

We use traffic related data extracted from loop detectors, allowing the detection of vehicles passing or arriving at certain points, positioned on a 2-mile section of the SR528 highway in Orlando, Florida covering both east and west bound traffic [1]. Speed, volume and occupancy were collected for each detector and aggregated for 1 minute. Figure 3.3 shows a google map snapshots of the chosen section of SR528. This section includes 2 on-ramps and 2 off-ramps.

Figure 3.3: SR528 Highway

For precisely recreating a real-world traffic scenario, the US Department of Transportation released micro-simulation guidelines. The development of the simulation model and model calibration were done in compliance with the standards established by the US Department of Transportation. Finally, the simulation model was assessed using data from actual traffic [12]. The traffic volume was used to calibrate the

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[1] City of Orlando, Florida, USA

study's parameters. In this work, Geoffrey E. Heavers (GEH) statistics were used to calibrate the model. This considers both the absolute value and the percentage difference. GEH determines the model's goodness of fit [15]. The formula to determine GEH is as follows:

$$GEH = \sqrt{\frac{2(V_{obs} - V_{sim})^2}{(V_{obs} + V_{sim})}} \tag{3.10}$$

The traffic volumes of the detectors in the simulation are represented by V_{sim}. If the value of GEH is less than 4 for the total number of cars in all of the links, i.e., on-ramps, off-ramps and freeways in between, then the simulation model is judged to be a good fit [12]. For this investigation, the GEH value for the calibrated parameters was 1.26. This value indicates that the simulated vehicle volume matches the actual field volume. Table 3.1 provides the calibrated values thank to SUMO calibrators, where *Tau* represents the desired headway and *Sigma* reflects real-world variations in the driver's behavior.

Table 3.1: SUMO Calibration Process

Parameters	**Default Value**	**Range**	**Calibrated Value**
Acceleration (m/s^2)	2.6	2.6-5.6	4.5
Deceleration (m/s^2)	4.5	4.5-7.5	6.5
Tau	1	1-1.5	1.3
Sigma	0.5	0.1-0.5	0.2

In order to verify the simulation model, we use the average field speed aggregated for 1 minute from the detectors. The absolute difference between simulation speed and field speed must be less than 5mi/h in 85% of the cases [50]. In 93% of the situations in our model, the absolute speed difference between the detectors' simulated average speed and field average speed was less than 5 mi/h. This result implies that the developed traffic simulation model is accurate and compatible with actual traffic conditions, supporting its validity.

3.3.3 Validation of the Formalized Traffic Safety Property

Since there is no direct link between the applied TCTs, i.e., TTC, SHW and SWV, was investigated before, the described and formalized traffic property in this chapter is considered a novelty in this work. However, the bidirectional relation between these two indicators needs to be proven and its accuracy must be investigated through the monitoring of a real-life traffic flow and analysing the performance measures highlighted by the property.

Using the traffic indicators extracted during this period, we will conduct our analysis based on their computed values in every time step and observe the resulting events and consequences of the situation. With a big number of vehicles in play, a platoon of vehicles is identified by pinpointing the leading vehicle and from then on the rest of the following vehicles will be identified. Using SUMO, every vehicle is represented by a vehicle ID making it easy to identify vehicles in a traffic flow. Furthermore, every following vehicle is accompanied with its ID, speed and acceleration, in addition to the leading vehicle ID as well as its speed and acceleration.

During this simulation, a list of traffic safety measures will be extracted, such as TTC and space headway. However, the shockwave indicator, i.e., shockwave speed, is not directly determined using the simulation but manually computed. In fact, for an accurate analysis of the shockwave occurence, a range of vehicles should be considered when calculating the speed. Seeing that the used dataset is collected in a highway implicating different lanes are in use, a manual treatment of the data is mandatory in order to identify a series of leading-following vehicles using the same lane during the congestion period. After the identification of a vehicle platooning, the shockwave speed is computed over a range of vehicles in this platoon to determine the existence or non-existence of a shockwave based on the defined thresholds of the speed.

Validation Results

As described above, the identification of a vehicle platooning in the real life dataset allows us to analyse the extracted indicators values along with determining the shockwave speed. Table 3.2 depicts the vehicles' IDs, speeds, space headways and TTCs extracted values forming the platoon, where each vehicle has its own leader and following vehicle. Based on Equation 3.7, the shockwave speed is computed for the introduced platoon while taking the vehicle with the ID "car432.28" as the leading vehicle and "car447.8" as the vehicle at the end of the presented platoon. Using the extracted parameters values, the shockwave speed is computed by replacing the parameters with their values in Equation 3.1 as follows:

$$SWV_{speed} = \frac{\frac{v_1}{d_1} - \frac{v_{15}}{d_{15}}}{\frac{1}{d_1} - \frac{1}{d_{15}}} = \frac{\frac{27.817}{38.631} - \frac{24.181}{36.581}}{\frac{1}{38.631} - \frac{1}{36.581}} = \text{-30 m/s} < 0$$

Per the definition of the property in Theorem 3.1, vehicles registering a TTC < 3 and a SHW < $\frac{1}{k}$, i.e., (SHW < 58.823 m), are involved in a traffic conflict. The computed shockwave speed in this case indicates the occurrence of a shockwave that is propagating downstream based on its negative sign. Moreover, analysing the TTC and space headway values stated in the Table 3.2, it is clear that the thresholds of both indicators are violated by most vehicles in the platoon which validates the formalized property.

Table 3.2: Validation of Extracted Data for a Vehicle Platooning

Time (s)	**Vehicle ID**	**Vehicle Speed (m/s)**	**Space Headway (m)**	**TTC (s)**
3600	**car432.28**	27.817	-	
3600	car446.0	24.877	38.631	10.69
3600	car447.0	25.137	35.359	0.95
3600	car450.4	26.133	117.046	84.24
3600	car446.1	26.592	36.678	0.9
3600	car450.3	26.522	36.576	0.9
3600	car450.2	26.888	36.825	1.15
3600	car447.4	25.273	37.256	1.15
3600	car446.2	23.204	38.181	0.75
3600	car446.3	22.424	45.604	0.75
3600	car446.10	23.209	39.961	16.78
3600	car446.6	23.793	31.661	0.57
3600	car450.6	23.792	34.183	0.57
3600	car447.7	24.582	35.852	1.21
3600	**car447.8**	24.181	34.647	1.21
3600	car450.7	24.545	36.581	0.73

The purpose of the validation is to prove the concreteness of the traffic safety property by studying a real-life platoon extracted from a calibrated real-life dataset. Analysing this set of vehicles and their respectful TTC and SHW aims to validate the formalized bidirectional relation between TTC, SHW and SWV. In fact, the shockwave speed of this platoon is calculated as mentioned above and the computed SWV confirms the existence of a shockwave that is further validated by the values of TTC and SHW presented in Table 3.2.

3.4 Summary

In this chapter, we provided a detailed description of a transportation safety property. As depicted by Figure 3.1, the proposed methodology describes the applied traffic conflict indicators, i.e, TTC, SHW and SWV as well as their formalization thanks to the use of differential dynamic logic (dL) as the specification and verification logic. Subsequently, we formally define the TTC, SHW and SWV relation using the same specification logic. Exploiting the mathematical modeling of the introduced TCTs along with the vehicles' dynamics, the defined safety property has been formally verified using the KeYmaera theorem prover.

From a transportation perspective, the bidirectional induction between TTC, SHW and SWV has to be confirmed by proving the existence of the phenomenon in real-life traffic. We conducted this validation using the traffic simulator SUMO over a real-life traffic dataset containing congestion areas. The main goal was to observe the simulation and extract the data about a platooning of vehicles presenting small TTC and SHW or a shockwave speed within the identified bounds confirming the existing of shockwaves. In the next chapter, we propose a direct application of the TTC-SHW-SWV safety property over the current real-life traffic dataset. Thereby, a framework using SUMO as a traffic simulation tool and Mathematica as an automated decision procedure, is developed. This integration lays the foundation to implementing an adaptive traffic management process that is exploited to reduce traffic conflicts.

Chapter 4

Case Study: Adaptive Traffic Management System

In general, Adaptive Cruise Control (ACC) systems automatically adapt the speed of vehicles to the behavior of other vehicles [49]. One of the notable early applications of ACC is the Automated Highway System PATH project [30] developed by the University of California that aims to organize the movements of vehicles to maximize the capacity and safety of traffic flow. Unlike ACC, in this chapter we implement an adaptive traffic management system that aims to automatically update vehicles' speeds in traffic conflicts. To this end, we apply our proposed traffic safety property over a real life traffic dataset in order to demonstrate the impact of the property and its violation on the vehicles' dynamics during traffic conflicts.

4.1 Methodology of Adaptive Traffic Management System

Unlike KeYmaera, Mathematica provides the appropriate API (Application Programming Interface) to be seamlessly integrated with SUMO. Mathematica's API allows a flexible connection with other tools and the launching of multiple sessions such as a Python session. The traffic simulator SUMO must be provided first with a calibrated dataset. This dataset is a real-life set of a traffic flow extracted from the sensors present in certain locations (highways, intersections, etc.). Upon the traffic simulation, the vehicles' dynamics are generated by SUMO specifying the vehicles speeds, accelerations, space headways, TTCs and leader vehicles if they exist.

We start by expressing the TTC-SWV traffic safety property, explored in Chapter 3, in the Wolfram language of Mathematica [39]. The safety conditions introduced by this property, i.e., TTC < 3 and SHW $< \frac{1}{k}$, are checked for every vehicle in the dataset to identify the vehicles violating them. We propose an adaptive traffic management system that aims to reduce traffic conflicts by controlling the violating vehicles' speeds. This system is established by defining a speed adaptation process in Mathematica in order to modify these vehicles' speeds by assigning a deceleration rate to which the vehicle must abide by reducing its speed gradually. The speed adaptation control process helps in avoiding multiple critical situations by controlling vehicles' speeds when the safety traffic rule is violated.

Figure 4.1 illustrates the integration of the traffic simulator SUMO with the computer algebra system Mathematica. Once the safety traffic property is violated, the adaptive traffic management system is executed and the speed update is introduced to SUMO in order for the traffic to adapt to the new changes. Once the speed adaptation process is done, the modified parameter is used to run the simulation again and update the traffic flow accordingly. In this chapter, we also provide a formalization of the speed adaptation process in dL and ensure its soundness by conducting a formal proof using KeYmaera.

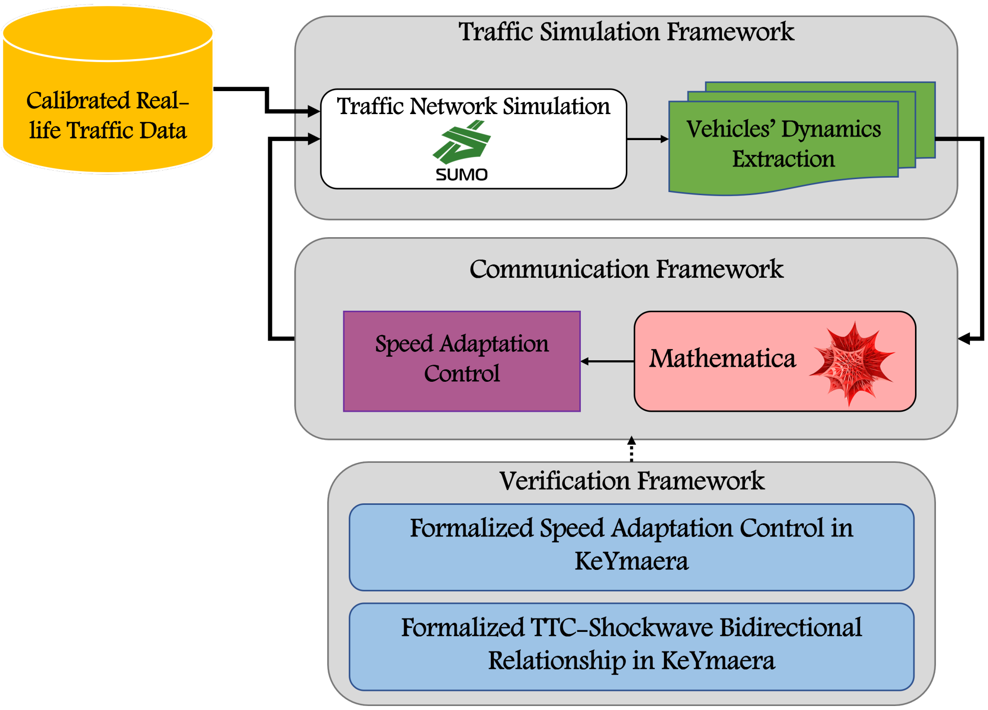

Figure 4.1: Methodology of Adaptive Traffic Management System

4.2 Adaptive Traffic Management System

As defined in Chapter 3, the verification of the traffic safety property depicting the bidirectional relation between TTC, SHW and SWV was a necessary step to be able to conduct this case study. As a matter of fact, we could not proceed without formally proving the defined property described in Theorem 3.1. The formal proof conducted in KeYmaera incorporates the proposed property along with the relation linking the described traffic safety indicators. For a TTC and a SHW below their respectful thresholds, i.e., 3 and $1/k$ with k being the density of the traffic flow, the occurrence of shockwaves is confirmed based on the bounds set for the shockwave speed (SWV), i.e., SWV < 0 or $0 <$ SWV < 7, at the microscopic level. This property is studied for the defined dataset in Chapter 3. First, we identify the vehicles falling under the mentioned constraints, in this case, we prioritize the vehicles belonging to a platoon where a slight modification of the vehicles dynamics are automatically reflected by the following vehicles as depicted by Figure 4.2. Afterwards, a speed adaptation process is automatically conducted in order to control the speed of the chosen vehicle in order to adapt to the traffic flow at hand and avoid future conflicts. In this context, the proposed speed adaptation control process is formalized in the dL language and formally proved using KeYmaera to ensure its soundness and accuracy.

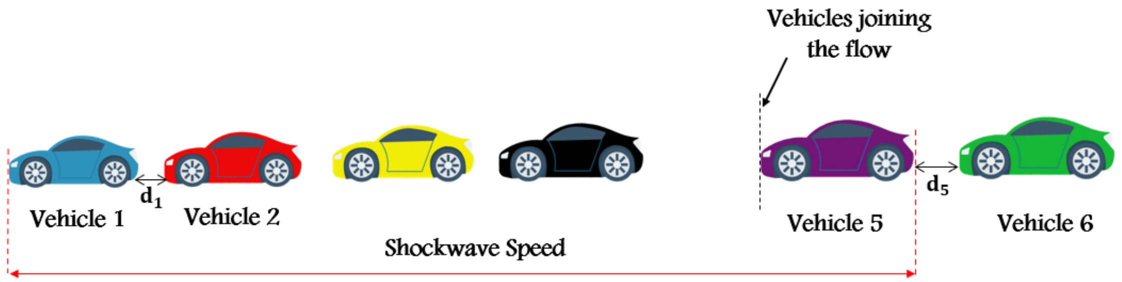

Figure 4.2: Illustration of Vehicle Platooning

The integrated speed feedback process aims to update the vehicle's dynamics, e.g., speed, in order to reach a target speed. Towards achieving this goal, this process is executed whenever one of the conditions of the property is violated and given by:

```
Speed Adaptation Control  ≡
                    if((SHW $\geq \frac{1}{k}$) ∨ (TTC > 3))
                    then
                       (∀ C i.  a(i) := A)
                    else
                       if((SHW < $\frac{1}{k}$) ∧ (TTC ≤ 3))
                       then
                          (∀ C i.  a(i) := -D)
                       else
                          (∀ C i.  a(i) := 0)
                       fi;
                       if((SWV < 0) ∨ (0 ≤ SWV ≤ 7))
                       then
                          (∀ C i.  a(i) := -D)
                       else
                          (∀ C i.  a(i) := A)
                       fi
                    fi
```

where the property evaluation is carried out for all vehicles in the simulation thanks to the use of the universal quantifier (∀) and $a(i)$ represents the acceleration of vehicle i, while A and D are two positive variables representing deceleration rates assigned to the vehicles.

Based on the formalization of the speed process depicted, the identified vehicle is evaluated according to its TTC and SHW values. If these values exceed the specified thresholds, the vehicle should decelerate accordingly in order to avoid conflicts and this is translated by assigning a deceleration rate to the vehicle in question in order to reduce its speed. However, if the computed values of the traffic safety indicators, i.e., TTC and SHW, are not violated, then we choose either to maintain the current vehicle's speed or allow it to accelerate. For instance, if a TTC is less than 3 or SHW is less than $1/k$, because of the speed adaptation loop, the speed is decreased by assigning a negative acceleration value to the vehicle, i.e., forcing the vehicle to decelerate, in order to adapt to the traffic flow and avoiding the formation of shockwaves. However, if the shockwave speed is within the bounds confirming the occurrence of a shockwave, then the taken decision is to decrease the speed for new vehicles joining the queue, otherwise, the vehicle maintains its current speed. Thanks to the ongoing traffic simulation, we monitor the changes in the traffic flow caused by the speed variation of vehicles violating the traffic safety property caused by external traffic factors.

4.3 SUMO-Mathematica Integration

Mathematica is chosen as a semi-formal verification tool in this case study thanks to its built-in library for theorem proving. Furthermore, it offers the advantage of interfacing with other tools that facilitates data exchange as well as sequential execution of certain tasks using different tools in one run. The specificity of this study appears in integrating Mathematica with SUMO. On one hand, a Python session is launched in Mathematica, along with defining the property and the speed adaptation control in the Wolfram Language.

```
psession = StartExternalSession["Python"]
ExternalEvaluate[psession, File["PATH"]]
```

On the other hand, the Wolfram Client Library for Python is exploited in SUMO, making it possible to call Wolfram language programs and execute them during the traffic simulation.

```
from wolframclient.evaluation import WolframLanguage-
Session
session = WolframLanguageSession(’WolfraKernal PATH’)
from wolframclient.language import wl
```

In order to reduce traffic conflicts and avoid future crashes, the introduced adaptive traffic management system is established through a set of steps described as follows:

1. Provided with a calibrated real-life dataset, the traffic simulation is ran through SUMO, where the vehicles dynamics at every time step along with the IDs of their leader vehicles are specified. This information is the key to proceed by introducing the extracted values to Mathematica. In fact, the values extraction is rendered possible due to the function defined in Mathematica, i.e., *ExternalValue*, that returns the value of the specified parameter from an external evaluation session. For instance, *sumoTTC* and *spaceHeadway* represent the Time-To-Collision (TTC) and Space Headway (SHW) between the current vehicle and its predecessor, respectively. As for *egoSpeed*, *veh*, and *leader*, they represent the speed, vehicle ID and leader vehicle ID of the current vehicle, respectively, in the traffic simulation.

   ```
   TTC:=ExternalValue[psession,"sumoTTC"];
   SHW:=ExternalValue[psession,"spaceHeadway"];
   Initial_Speed:=ExternalValue[psession,"egoSpeed"];
   vehID:=ExternalValue[psession,"veh"];
   leaderID:=ExternalValue[psession,"leader"];
   ```

2. After launching the Mathematica session, the Wolfram language program is executed and the vehicles violating the property are identified. Subsequently, based on the vehicles dynamics, the speed value is either updated or maintained as is. In the case where the vehicle is found to be violating the property, a new speed value is assigned to the vehicle.

3. After assigning the new speed value, the Mathematica session is terminated and the Python session for SUMO resumes its execution by reading the new assigned speed and updating the vehicle’s dynamics dynamically.

4. Consequently, the vehicles dynamics are extracted through SUMO, where the change made is prominent. After analysing the data, we notice the impact of the speed variation on the traffic flow, most importantly on the vehicles belonging to the platoon.

4.4 Results and Discussion

In the presence of a dataset of calibrated real-life traffic data, the observation of the changes impacting the traffic flow is not straightforward due to the large number of vehicles. For simplicity, we apply the safety property integrating a speed adaptation process on a platoon of vehicles. By violating the verified property, two samples of platoons are identified and the initial vehicles dynamics are extracted. For every vehicle in the platoon identified by a unique ID, its speed and acceleration are specified. Furthermore, the leader ID of the vehicle in question along with its speed are also extracted. In addition to these dynamics, the traffic safety indicators TTC and Space Headway, are also extracted for every two consecutive vehicles in the platoon.

Based on the provided dataset describing a real-life traffic flow on the highway SR528 in Florida, USA, the existence of congested areas is guaranteed, which explains the reduced values of TTC and SHW. Nevertheless, it is further confirmed by analysing the values of TTC and SHW, where most vehicles violate the property by reflecting a $\text{TTC} < 3$ and a $\text{SHW} < \frac{1}{k}$ as shown by Table 3.2. Moreover, computing the shockwave speed of the specified platoon, SWV_{speed}, results in a value less than 7 m/s that is equal to 1.39 m/s proving the occurrence of shockwaves. Starting the traffic simulation using SUMO, the first identified vehicle to be violating the property in the platoon undergoes the speed adaptation process. The latter consists on assigning a new speed value to make the vehicle in question adapt to the traffic flow and reduce the number of traffic conflicts by taking earlier actions. The vehicle in question decelerates smoothly for a time duration of 3 seconds until it reaches the target speed. Meanwhile, the applied deceleration impacts the behavior of vehicles in the platoon by making them adjust their speeds to the situation gradually. We conduct this process over two platoons of vehicles and provide the results by extracting the charts of the acceleration, Space Headway (SHW) and Time-To-Collision (TTC) of the original platoons versus the updated platoons.

We study the first identified platoon over a range of 4 seconds, i.e., from the time step 3604 to 3608, for 5 vehicles forming the platoon. The vehicle "car497.31", violating the property with a TTC = 2.9 (TTC < 3), is spotted at time step 3605 as shown by Figure 4.7. Consequently, time step 3605 represents the instant the speed change taking place.

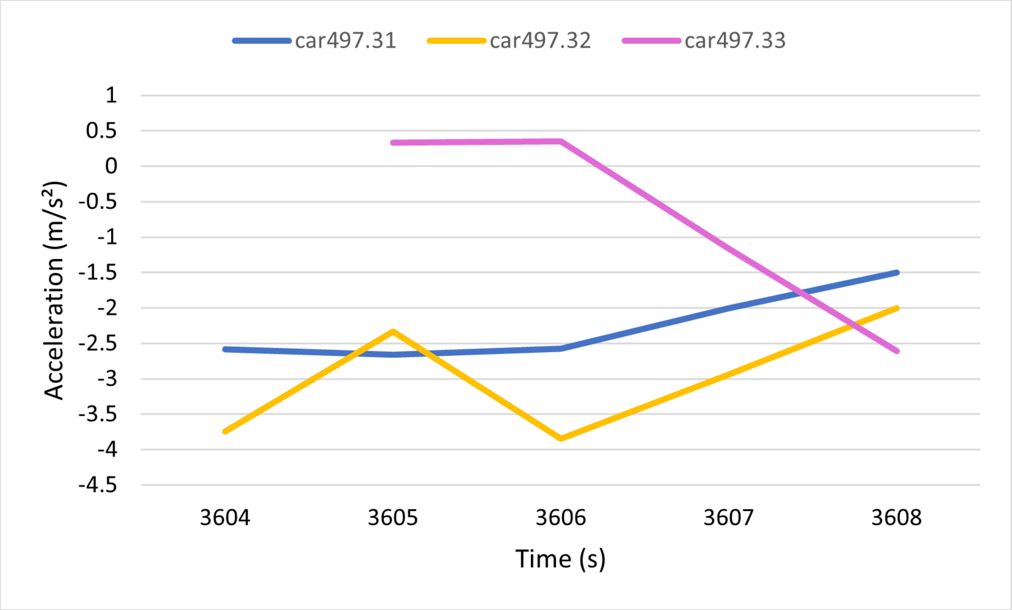

Figure 4.3: Acceleration Profile Computed for the Original Platoon

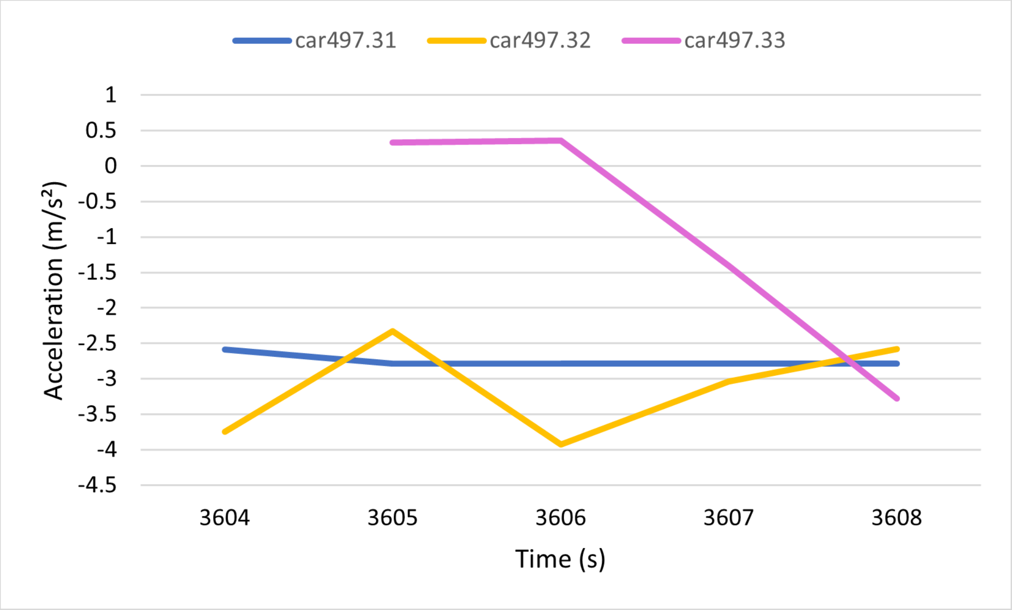

Figure 4.4: Acceleration Profile Computed for the Updated Platoon

The impact of the change is reflected by comparing the acceleration profile in Figure 4.3 corresponding to the original platoon with the acceleration profile given by Figure 4.4 representing the updated platoon after the speed update. The two following vehicles of “car497.31”, i.e., “car497.32” and “car497.33”, respectively, present a change of behavior compared to the original platoon and its acceleration profile. This change is reflected by a deceleration starting from the time step 3605, where a decrease rate of 9.7% is registered. At time step 3606, a decrease rate of 10.2% is noted, followed by a 63.1% decrease at 3607. At time step 3608, a decrease rate of 140.6% is achieved to reflect the deceleration of vehicles after the speed update compared to their acceleration before the speed update. This acceleration decrease illustrates the executed deceleration to reduce the vehicles speed and achieve a target speed due to the evaluated conflict (TTC < 3).

After analysing the acceleration profiles, a Space Headway (SHW) profile analysis is conducted to further study the the speed adaptation control impact on the considered platoon. In Figure 4.5, the SHW profile of the original platoon presented shows a decrease of the values of SHW of the three studied vehicles over time. Whereas, in Figure 4.6, the updated platoon SHW profile is given for the three vehicles including the vehicle “car497.31” that undergoes the speed update at time step 3605.

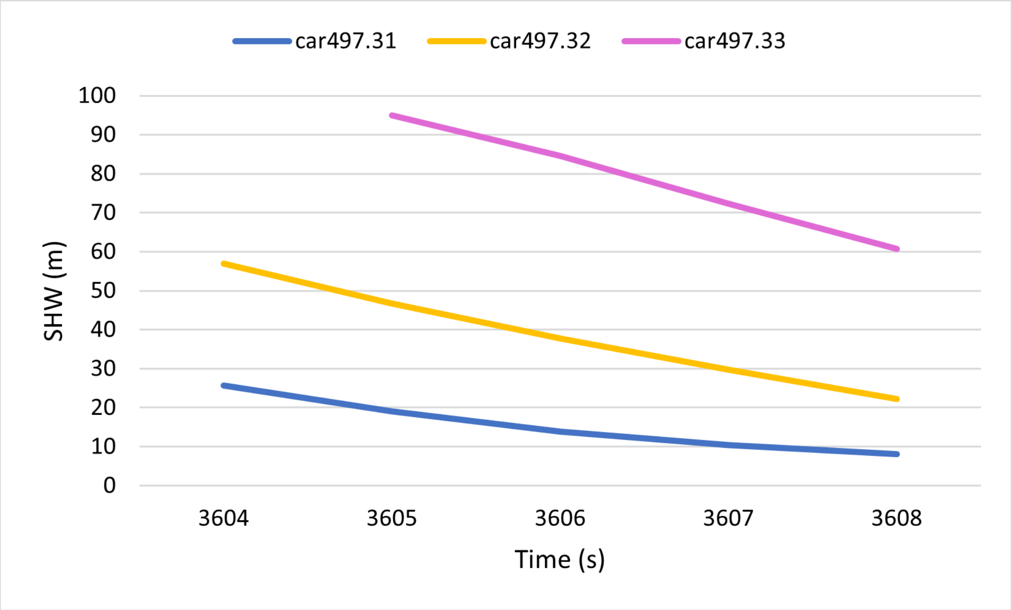

Figure 4.5: Space Headway Profile Computed for the original Platoon

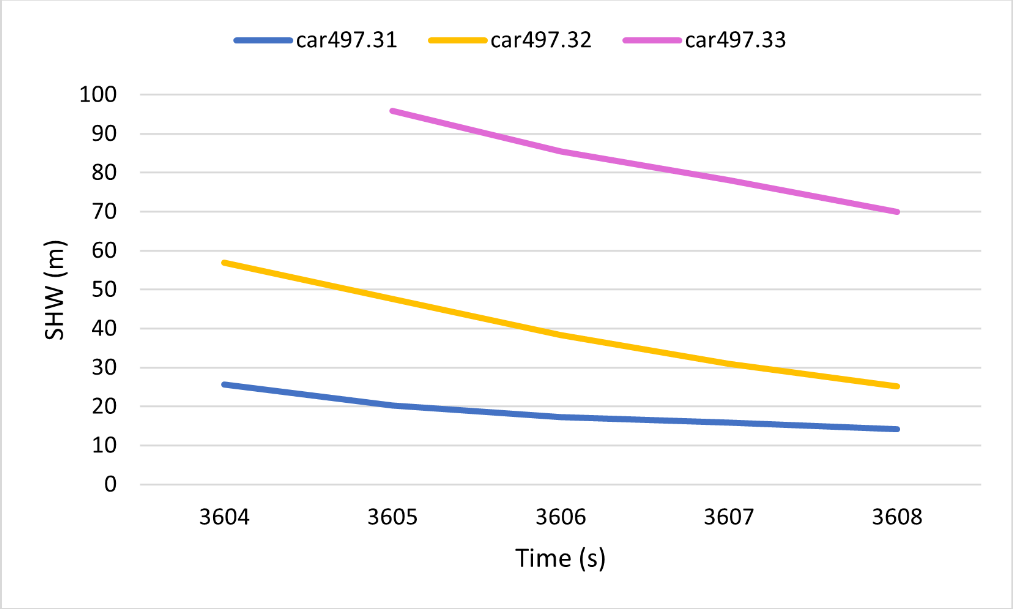

Figure 4.6: Space Headway Profile Computed for the Updated Platoon

The difference between the presented SHW profiles is visible in Figure 4.6, where the SHW values of each vehicle register a progressive increase for each time step compared to the original space headway values. For instance, at time step 3605 an increase rate of 8.8% is noted, where at time step 3606, the increase rate hit 27.8%. At time step 3607 and 3608, 66% and 102.7% are registered as increase rates of SHW values.

The studied speed update impact reaches the time to collision (TTC) values for the vehicles in the platoon, where a variation of TTC is noticed. The TTC profile for the platoon pre- and post- the speed update is given by Figures 4.7 and 4.8, respectively. Compared to the original TTC values, an increase is reflected by the new values thanks to the speed adaptation process. This change is reflected by the TTC profiles presented in Figures 4.7 and 4.8, where an increase rate of 1.2% is noted at time step 3605. At time step 3606, the increase reached 70%, where at time steps 3607 and 3608, the vehicles achieved a total of 188.9% and 265.8% increase in TTC values, respectively, which is the aim of the execution of the speed adaptation process when traffic conflicts are detected.

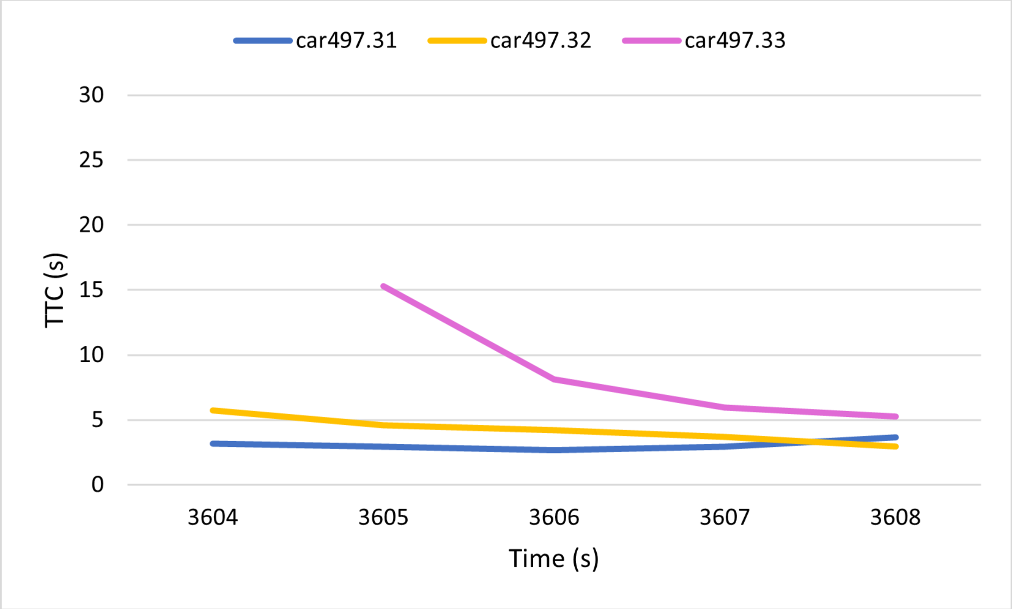

Figure 4.7: Time To Collision Profile Computed for the original Platoon

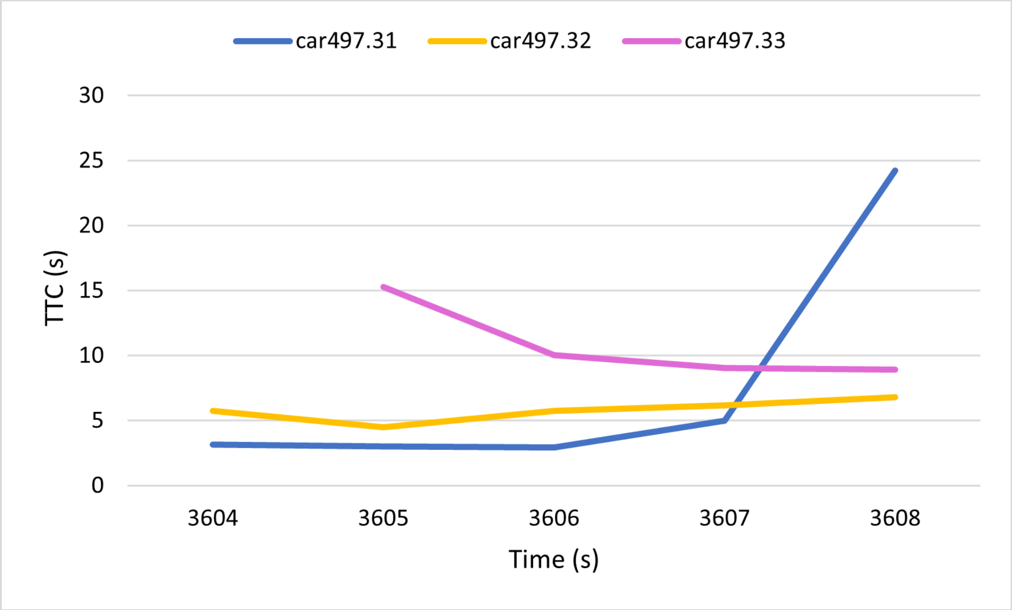

Figure 4.8: Time To Collision Profile Computed for the Updated Platoon

The second considered platoon consists of eight vehicles that are monitored over a range of five seconds, the violating vehicle, "car457.22" with a TTC < 3, is detected at the time step 3604. Focusing our study on the vehicles following the violating car, we provide the acceleration profile, space headway (SHW) profile and the time to collision (TTC) profile for the five following vehicles. The analysis of the provided profiles is conducted from time step 3604 to 3608, however, the repercussion of the speed update is observed starting from time step 3605.

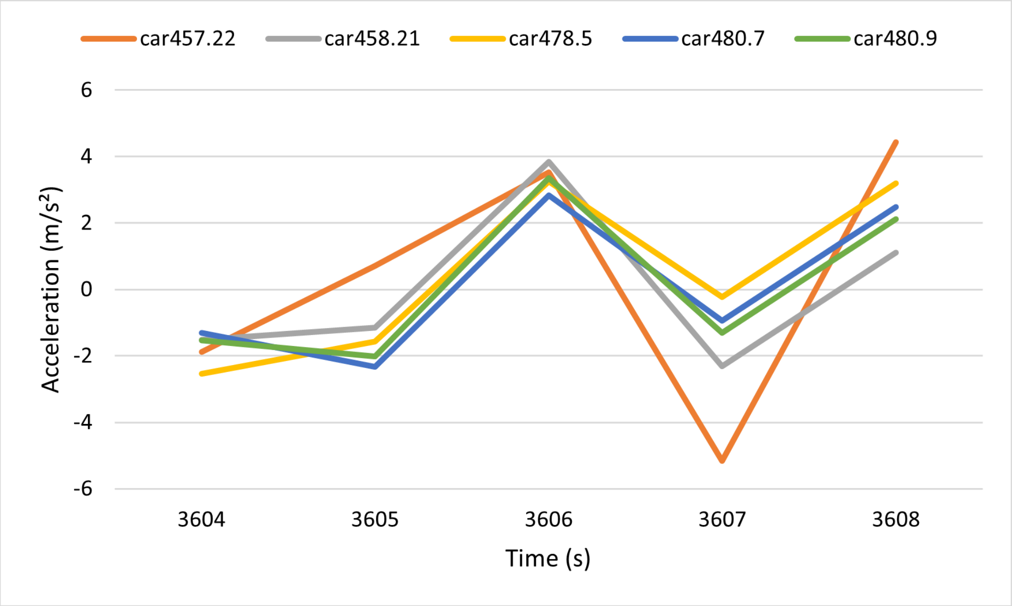

Figure 4.9: Acceleration Profile Computed for the Original Platoon

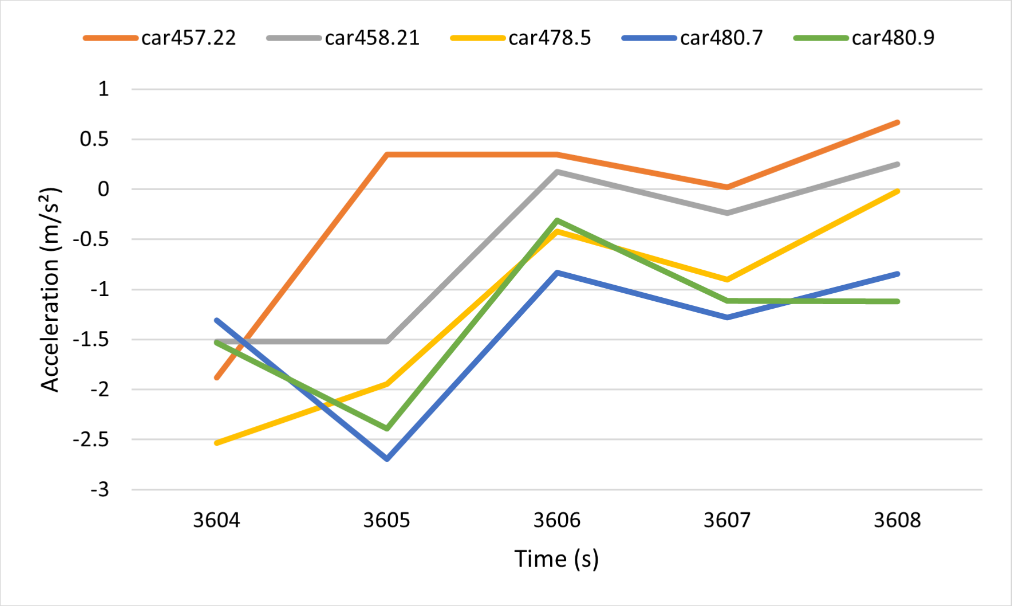

Figure 4.10: Acceleration Profile Computed for the Updated Platoon

In Figure 4.9, the acceleration profile for the vehicles in the platoon before the speed update is presented. For instance, the acceleration of vehicle “car457.22” over the time is unstable with sudden peaks and drops according to the traffic situation, where the acceleration abruptly increases above 4 m/s^2 at time step 3608. However, analysing the behavior of the same vehicle, i.e., “car457.22”, after the speed update was conducted, reflects a smooth and small increase of acceleration that it is kept below the original values for each time step. As confirmed by Figure 4.10, the maximum value reached is below 1 m/s^2. Furthermore, the observed ramification on the acceleration is further confirmed by the computed percentage decrease rate at every time step. The speed adaptation impact is observed progressively over time which is reflected by the computed percentage decrease rates. For instance, a percentage decrease rate of 51.6% is registered at time step 3605, whereas a 243.3% decrease rate is achieved at time step 3606. Furthermore, a decrease rate of 327.7% and 269.7% are registered at time steps 3607 and 3608, respectively.

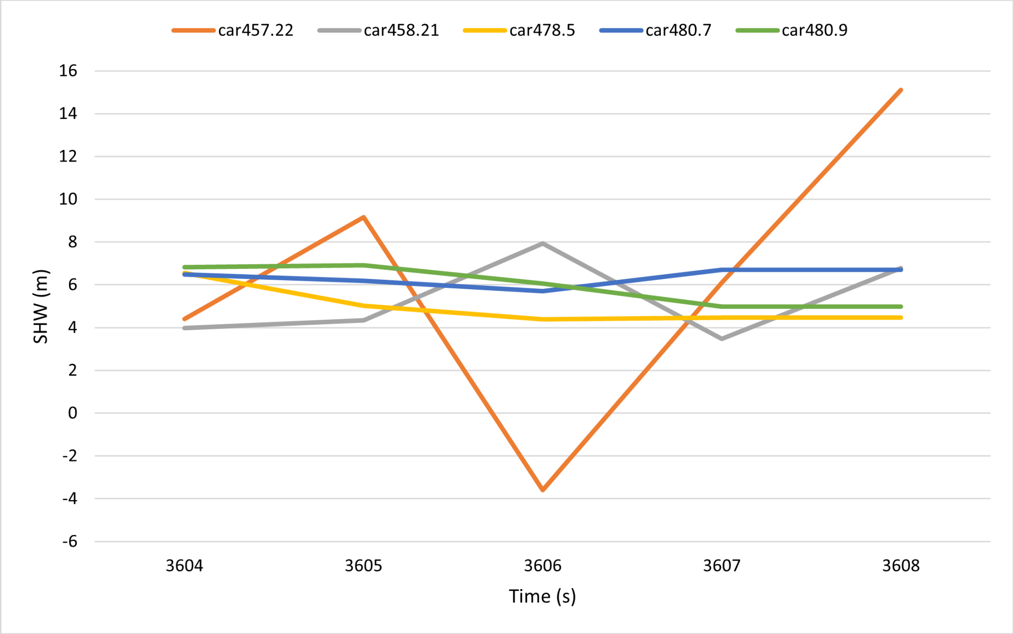

Figure 4.11: Space Headway Profile Computed for the Original Platoon

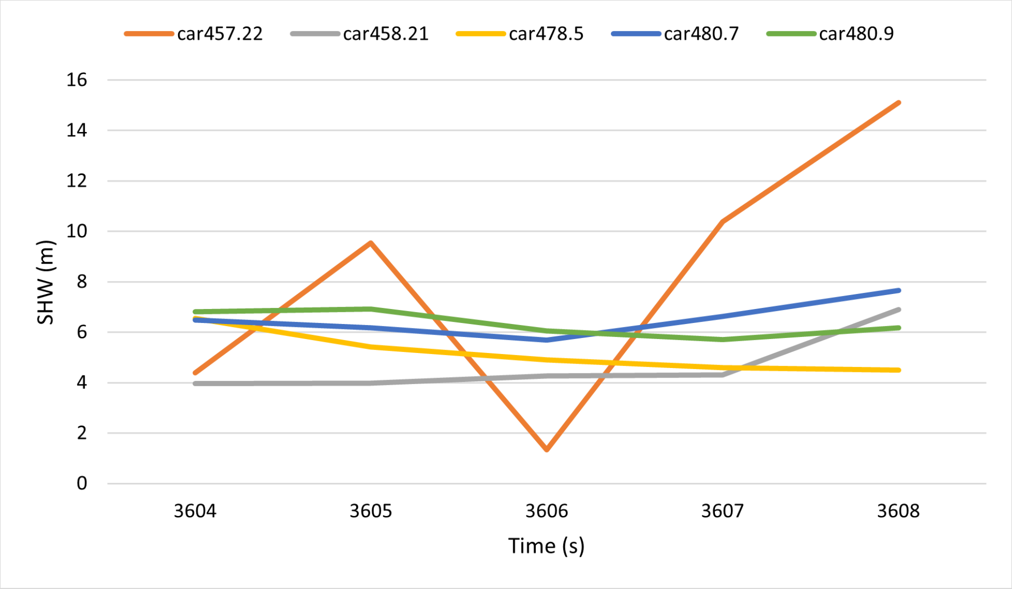

Figure 4.12: Space Headway Profile Computed for the Updated Platoon

Figures 4.11 and 4.12 depict the space headway (SHW) profile before and after conducting the speed adaptation process, respectively. The figures highlight the increase of the SHW values for every considered vehicle in the platoon thanks to the speed reduction. For instance, the violating vehicle "car457.22", as shown by Figure 4.13, presents a negative value for the space headway below -2 m meaning a crash that took place at time step 3606 as shown in Figure 4.11. However, applying the speed change at time step 3605 managed to prevent the occurrence of the crash by enhancing the SHW to be 1.3 m as reported by Figure 4.12. This improvement is extended to the other vehicles in the platoon and confirmed by the percentage increase rate computed for every time step. At time step 3605, we noted a total of 3.5% increase of SHW values. Furthermore, we achieved a 102.4% increase at time step 3606 and a 95.5% increase of SHW at time step 3607. At the last time step, i.e., 3608, a 16.9% increase rate is realized.

After inspecting the ramifications of speed adaptation control process on SHW values, it is interesting to study the repercussion of executing the speed update process on TTC values of the vehicles in the platoon.

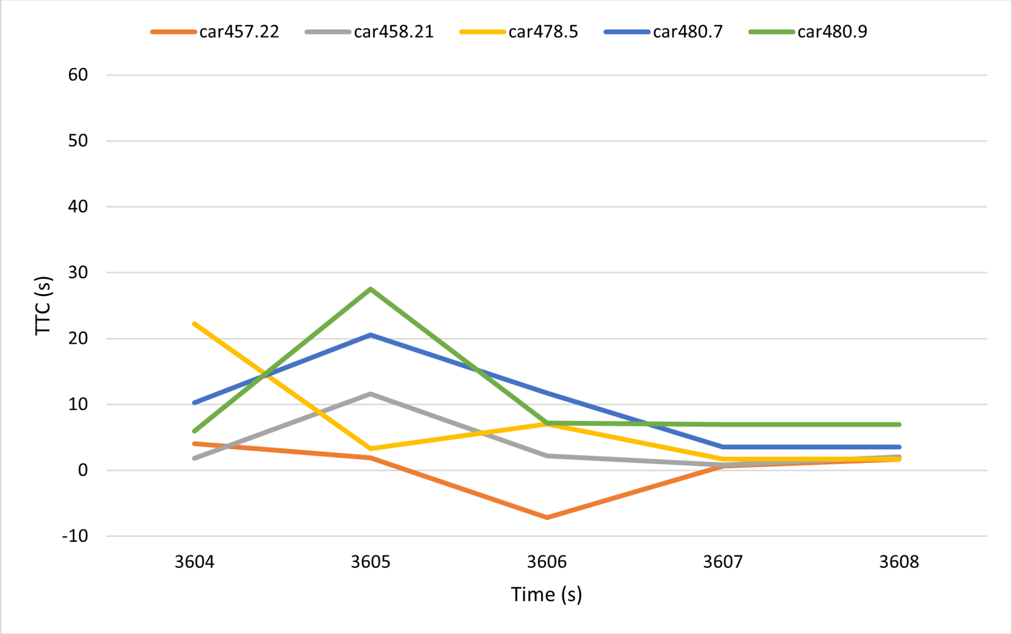

Figure 4.13: Time To Collision Profile Computed for the Original Platoon

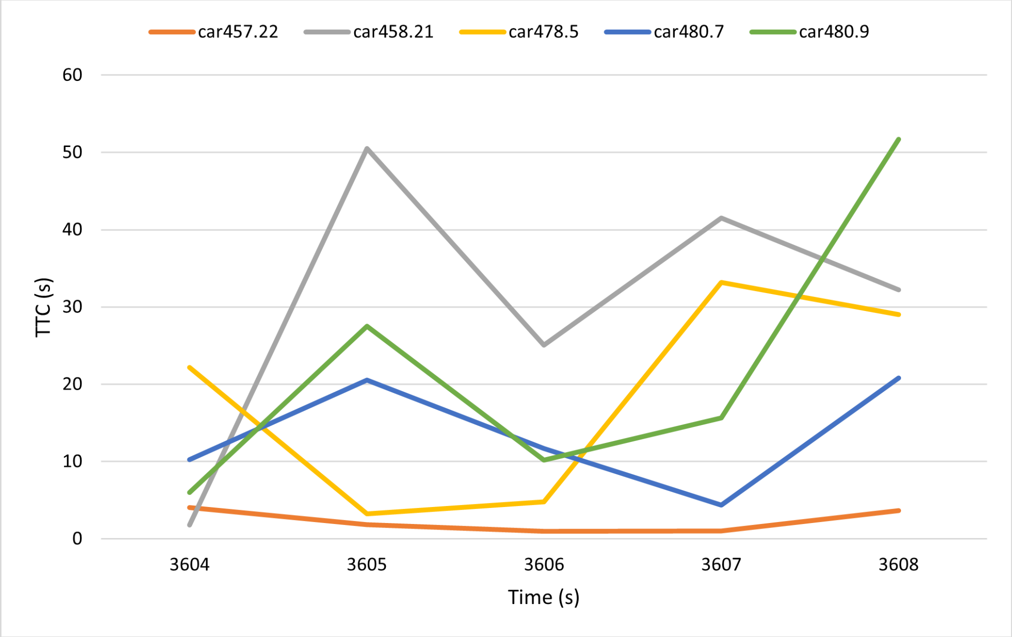

Figure 4.14: Time To Collision Profile Computed for the Updated Platoon

To achieve this goal, we provide the TTC profiles in Figures 4.13 and 4.14 where the outcome of the speed adaptation process is observed. The obtained increase of the vehicles TTC over time is clear when comparing the TTC profiles before and after the adaptation process. Furthermore, computing the percentage increase rate of TTC for every time step, we register a 150% increase in the values of TTC at time step 3605. Moreover, an increase rate of 250% is achieved at time step 3606, in addition to a 275.5% and a 290.8% increase achieved at time steps 3607 and 3608, respectively.

In summary, the integration of SUMO with Mathematica proved efficient in identifying traffic conflicts through applying the TTC-SWV traffic safety property over a real-life traffic data. Once the violating vehicle is identified, the corresponding platoon of vehicles is put under the microscope and a speed adaptation process is executed to update the speed of the vehicle in question. However, this update is interpreted by analysing the main performance measures in the traffic property, i.e., TTC and SHW. The analysis is done by listing the TTC and SHW profiles over a specific range of time of two identified platoons where a conflict is spotted. Based on the performed analysis of the profiles, the results are tangible and a clear improvement of the TTC and SHW values is noted which reflects a reduction in the number of conflicts once the speed adaptation process is carried. As a summary, Table 4.1 is provided to highlight the impact of the implemented adaptive traffic management system built on the combination of the introduced traffic safety property with the traffic simulator SUMO. As reflected by Table 4.1, the overall improvement is very prominent by registering up to 265.8% and 102.7% increase of TTC and SHW values for platoon 1. Furthermore, an improvement of TTC of 290.8% is registered for platoon 2, accompanied by up to 102.4% increase in SHW values. This increase stipulates that by assigning new speed values to violating vehicles, we achieved longer TTC and longer distances separating consecutive vehicles in the platoons. Consequently, the implemented adaptive traffic management process is successful in reducing the number of traffic conflicts induced by the occurrence of shockwaves.

Table 4.1: Improvement Statistics of the Adaptive Traffic Management System

Platoon	**Time Step**	**Improvement Rate**		
		Acc	**SHW**	**TTC**
Platoon 1	3605	9.7%	8.8%	1.2%
	3606	10.2%	27.8%	70%
	3607	63.1%	66%	188.9%
	3608	140.6%	102.7%	265.8%
Platoon 2	3605	51.6%	3.5%	150%
	3606	243.3%	102.4%	250%
	3607	327.7%	95.5%	275.5%
	3608	269.7%	16.9%	290.8%

4.5 Summary

In this chapter, we proposed a direct application of the defined safety property in Chapter 3 through the integration of SUMO with Mathematica. This integration of the two tools is conducted to demonstrate the practical case study of the verified property over a real-life dataset. It represents a proof of concept of the feasibility of the proposed adaptive traffic management process. The proposed approach in this chapter is based on simulating a real-life dataset and monitoring the computed TTC and SHW for consecutive vehicles belonging to the same platoon. Once a violation of the mentioned traffic indicators is noted, a speed adaptation process is executed based on the integration of Mathematica with SUMO. This speed update is conducted over a vehicle in the platoon violating the safety property. Re-running the simulation, the impact of the speed update is reflected by the speed reduction of the following vehicles to adapt their driving to the situation at hand.

The integration of the two proposed tools in this work offered the flexibility of monitoring vehicles dynamics and their registered traffic safety indicators at each time step of the traffic simulation. Furthermore, the analysis of the extracted values is carried out using the formalized safety property, where a feedback process is executed in case a property violation is detected. This feedback process is a speed adaptation loop where the vehicles violating the thresholds of TTC and SHW are assigned new speed values to be reached smoothly in order to avoid upcoming conflicts and reduce their occurrence. Using the provided traffic dataset for the SR528 highway, we achieved our goal by reaching increased SHW and TTC values that reduce traffic conflicts by reducing the occurrence of shockwaves. The described case study represents a successful demonstration of the efficiency of the defined safety traffic property. Furthermore, this integration of tools extends the same treatment to other real-life calibrated datasets allowing the extraction, analysis and control of vehicles' dynamics when a safety property violation is detected.

Chapter 5

Conclusion and Future Work

5.1 Conclusion

Urban mobility is witnessing a growing reliance on advanced driver-assistance systems (ADAS) for traffic safety mitigation. Yet, ADAS and the anticipated deployment of driver-less vehicles expose the traffic network to an unprecedented level of technologies that might not be mature enough to deal with the complex nature of the traffic culture. The pre-mature reliance on advanced technologies that are deployed can have severe consequences on road safety. To study the requirements for safe driving and guarantee the safety of vehicle occupants, we used formal verification based on automated theorem proving to prove or disapprove the correctness of the safety properties during interacting events.

In our work, we consider certain Traffic Conflict Techniques (TCTs), such as temporal and spatial proximity traffic indicators, in addition to speed-related, conflict severity and impact of the crash traffic safety indicators. These indicators are put to use by forming two safety traffic properties. The first defined property aims to formally analyse crashes and their severity level by studying Time-To-Collision (TTC), Extended Delta-V, deceleration rate along with the jerk profile and yaw rate indicators. We focus our study on the analysis of the driver's behavior based on his actions and their magnitude in a traffic conflict. For a crash-free traffic flow, a convenient evasive action should be applied at the right time and with the right intensity. As for the second property, it connects the occurrence of shockwaves to the variation of the values of TTC and space headway (SHW). A decrease in these two values below certain thresholds leads to a shockwave formation based on the computation of the shockwave speed indicator. Furthermore, it is noticeable that a decrease in the value of TTC and SHW can be caused by a shockwave that has occurred. For instance, an existing shockwave that is propagating downstream impacts the reaction of vehicles recently joining the flow by forcing them to brake abruptly which leads to reduced TTC and SHW. This bidirectional relation was explored and validated in this work using a real-life dataset.

The application of formal verification in our work appears in the formalization of the defined traffic safety properties using differential dynamic logic (dL) and their verification using an automated theorem proving tool, namely KeYmaera. Furthermore, the two formalized properties were successfully verified ensuring their soundness and accuracy. The analysis of the TCTs used allows the identification of future traffic conflicts that may lead to accidents. Furthermore, these indicators can be exploited in generating feedback loops serving as management systems in order to report future conflicts for incoming vehicles. This would allow the vehicles sufficient time to adapt their dynamics for the situation coming ahead. To satisfy this purpose, a case study was detailed in Chapter 4 where a traffic simulator was exploited to simulate a calibrated real-life dataset. Afterwards, we extracted the vehicles dynamics at each time step and analysed them. According to the values of the performance measures, i.e., TTC and SHW, a speed adaptation process is executed via the computer algebra system Mathematica. The aim of this process is to identify the vehicles violating the traffic safety property defined in Chapter 3 and to assign new speed values to make the vehicles adapt better to the current traffic flow.

In summary, we were able to improve the safety of traffic transportation by providing reliable traffic management properties, in addition to a formal analysis of crashes severity levels based on the drivers behavior. In this context, the use of formal verification is crucial when it comes to ensuring the accuracy and the soundness of the proposed safety traffic properties. Furthermore, the integration of the SUMO traffic simulator with Mathematica, lays a foundation for further investigations to connect traffic simulators with formal verification tools. It is interesting to note that applying new acceleration values to certain vehicles falling under the restrictions specified by the property proves its efficiency by reducing the number of traffic conflicts and avoiding future conflicts.

5.2 Future Work

The safety of transportation will remain an unreachable goal unless it is seen from another perspective, for this, formal verification is essential to uncover the gaps of existing techniques and to prove the efficiency of emerging methods. The work presented in this thesis lays a foundation for future work in this field. Towards achieving this goal, we present a list of future tasks that can improve traffic safety:

- The proposed traffic safety properties in this work focus on car following models, side collisions and head-on collisions. However, it will be interesting to consider different traffic behaviors such as lane changing, weaving and different traffic road characteristics, e.g., roundabouts where side-swipe conflicts can be predominant.

- The proposed case study in this work aims to integrate two tools in order to establish a traffic management system to improve traffic safety. It represents a proof of concept about the feasibility of the idea. However, challenges were met during the implementation caused by the instability of the Wolfram Mathematica API. In order to optimize this work, Satisfiability Modulo Theories (SMT) solvers can be a convenient solver to be integrated with SUMO in order to achieve an accurate traffic management system.

- In this thesis, the delivered work is based on the analysis of the vehicles interactions by studying the drivers behaviors, especially in traffic conflicts. However, as a future work, we can expand this analysis to incorporate connectivity by considering autonomous and connected vehicles. By considering the intercommunication between these vehicles and the delays that come with it, the results of the formal analysis of traffic safety will be interesting to interpret.

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