

# 1 rewrite\_Rules\_Lemmas Theory

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**Parent Theories:** DFT\_Gates\_def\_PROB

## 1.1 Definitions

[[k\\_out\\_def](#)]

$\vdash \forall k L. \text{k\_out } k L = \{ s \mid s \subseteq \text{set } L \wedge (\text{CARD } s = k) \}$

[[k\\_out\\_n\\_gate\\_def](#)]

$\vdash \forall k L.$   
 $\text{k\_out\_n\_gate } k L =$   
 $\text{n\_OR}$   
 $(\text{MAP } (\lambda a. \text{n\_AND } (\text{SET\_TO\_LIST } a))$   
 $(\text{SET\_TO\_LIST } (\text{k\_out } k L)))$

[[n\\_AND\\_def](#)]

$\vdash \forall L. \text{n\_AND } L = \text{FOLDL } (\lambda a b. \text{D\_AND } a b) \text{ ALWAYS } L$

[[n\\_OR\\_def](#)]

$\vdash \forall L. \text{n\_OR } L = \text{FOLDL } (\lambda a b. \text{D\_OR } a b) \text{ NEVER } L$

[[n\\_PAND\\_def](#)]

$\vdash \forall L. \text{n\_PAND } L = \text{FOLDL } (\lambda a b. \text{P\_AND } a b) \text{ ALWAYS } L$

[[rv\\_gt0\\_def](#)]

$\vdash (\text{rv\_gt0 } [] \iff \text{T}) \wedge$   
 $\forall h t. \text{rv\_gt0 } (h :: t) \iff (\forall s. 0 \leq h s) \wedge \text{rv\_gt0 } t$

[[UNIONL\\_def](#)]

$\vdash (\text{UNIONL } [] = \{\}) \wedge \forall s ss. \text{UNIONL } (s :: ss) = s \cup \text{UNIONL } ss$

## 1.2 Theorems

[[ALL\\_DISTINCT\\_MAP\\_set](#)]

$\vdash \forall L. \text{ALL\_DISTINCT } L \Rightarrow \text{ALL\_DISTINCT } (\text{MAP } (\lambda a. \{ a \}) L)$

[[D\\_RIGHT\\_AND\\_OVER\\_OR](#)]

$\vdash \forall X Y Z. \text{D\_AND } (\text{D\_OR } X Y) Z = \text{D\_OR } (\text{D\_AND } Z X) (\text{D\_AND } Z Y)$

[[FINITE\\_IN\\_k\\_out](#)]

$\vdash \forall s k L. s \in \text{k\_out } k L \Rightarrow \text{FINITE } s$

[FINITE\_k\_out]

⊢ ∀L k. FINITE (k\_out k L)

[IN\_REST]

⊢ ∀x s. x ∈ REST s ⇔ x ∈ s ∧ x ≠ CHOICE s

[IN\_UNIONL]

⊢ ∀l v. v ∈ UNIONL l ⇔ ∃s. MEM s l ∧ v ∈ s

[k\_out\_CARD]

⊢ ∀s k L. s ∈ k\_out k L ⇒ (CARD s = k)

[k\_out\_n\_empty]

⊢ ∀k. k ≠ 0 ⇒ (k\_out k [] = {})

[k\_out\_n\_greater]

⊢ ∀k L. LENGTH L < k ⇒ (k\_out k L = {})

[k\_out\_of\_n\_gate\_OR]

⊢ ∀L. ALL\_DISTINCT L ∧ rv\_gt0 L ⇒ (k\_out\_n\_gate 1 L = n\_OR L)

[lem10]

⊢ ∀x y s. y s ≤ P\_AND x y s

[lem11]

⊢ ∀x y s. x s ≤ P\_AND x y s

[lem12]

⊢ ∀x y s L. rv\_gt0 (x::y::L) ⇒ y s ≤ n\_PAND (x::y::L) s

[lem8]

⊢ ∀x L. rv\_gt0 L ⇒ (n\_PAND (L ++ [x]) = P\_AND (n\_PAND L) x)

[lem9]

⊢ ∀y s L h. y s ≤ h s ⇒ y s ≤ FOLDL (λ a b. P\_AND a b) h L s

[LENGTH\_k\_out\_MEM]

⊢ ∀x k L. MEM x (SET\_TO\_LIST (k\_out k L)) ⇒ (CARD x = k)

[LIST\_APPEND\_middle]

⊢ ∀h L L1. L ++ [h] ++ L1 = L ++ h::L1

[MAP\_MEM]

⊢ ∀x f L. MEM x L ⇔ MEM {x} (MAP (λ a. {a}) L)

[MEM\_CHOICE\_k\_out]

$$\vdash \forall x. L. \begin{aligned} & \text{SING } x \wedge \text{MEM (CHOICE } x) \text{ } L \Rightarrow \\ & \text{MEM } x \text{ (SET\_TO\_LIST (k\_out 1 } L)) \end{aligned}$$

[MEM\_k\_out\_1\_SING]

$$\vdash \forall x. L. \text{MEM } x \text{ (SET\_TO\_LIST (k\_out 1 } L)) \Rightarrow \exists a. x = \{ a \}$$

[MEM\_k\_out\_CHOICE]

$$\vdash \forall x. L. \text{MEM } x \text{ (SET\_TO\_LIST (k\_out 1 } L)) \Rightarrow \text{MEM (CHOICE } x) \text{ } L$$

[MEM\_k\_out\_IN\_L]

$$\vdash \forall x. L. \text{MEM } x \text{ (SET\_TO\_LIST (k\_out 1 } L)) \Rightarrow x \in \text{k\_out 1 } L$$

[MEM\_k\_out\_MEM\_L]

$$\vdash \forall x. L. \begin{aligned} & \text{MEM } x \text{ (SET\_TO\_LIST (k\_out 1 } L)) \iff \\ & \text{MEM } x \text{ (MAP } (\lambda a. \{ a \}) \text{ } L) \end{aligned}$$

[MEM\_SING\_elem\_k\_out]

$$\vdash \forall a. L. \text{MEM } a \text{ } L \Rightarrow \text{MEM } \{ a \} \text{ (SET\_TO\_LIST (k\_out 1 } L))$$

[MEM\_SING\_k\_out\_equal]

$$\vdash \forall a. L. \text{MEM } a \text{ } L \iff \text{MEM } \{ a \} \text{ (SET\_TO\_LIST (k\_out 1 } L))$$

[MEM\_x\_SING]

$$\vdash \forall x. L. \text{MEM } x \text{ (MAP } (\lambda a. \{ a \}) \text{ } L) \Rightarrow \exists a. x = \{ a \}$$

[n\_AND\_APPEND]

$$\vdash \forall L_1. L_2. \begin{aligned} & \text{rv\_gt0 } (L_1 ++ L_2) \Rightarrow \\ & (\text{n\_AND } (L_1 ++ L_2) = \text{D\_AND } (\text{n\_AND } L_1) \text{ } (\text{n\_AND } L_2)) \end{aligned}$$

[n\_AND\_comm]

$$\vdash \forall L_1. L_2. \text{PERM } L_1 \text{ } L_2 \Rightarrow (\text{n\_AND } L_1 = \text{n\_AND } L_2)$$

[n\_AND\_GTO]

$$\vdash \forall L. \text{rv\_gt0 } L \Rightarrow \forall s. 0 \leq \text{n\_AND } L \text{ } s$$

[n\_AND\_SING]

$$\vdash \forall L. \text{rv\_gt0 } L \wedge (\text{LENGTH } L = 1) \Rightarrow (\text{n\_AND } L = \text{HD } L)$$

[n\_AND\_SING\_set]

$$\vdash \forall s. \begin{aligned} & \text{rv\_gt0 (SET\_TO\_LIST } s) \wedge (\text{CARD } s = 1) \wedge \text{FINITE } s \Rightarrow \\ & \exists x. x \in s \wedge (\text{n\_AND (SET\_TO\_LIST } s) = x) \end{aligned}$$

[n\_OR\_APPEND]

$\vdash \forall L_1 \ L_2. \ n\_OR \ (L_1 \ ++ \ L_2) = D\_OR \ (n\_OR \ L_1) \ (n\_OR \ L_2)$

[n\_OR\_comm]

$\vdash \forall L_1 \ L_2. \ PERM \ L_1 \ L_2 \Rightarrow (n\_OR \ L_1 = n\_OR \ L_2)$

[n\_OR\_GTO]

$\vdash \forall L_1. \ rv\_gt0 \ L_1 \Rightarrow \forall s. \ 0 \leq n\_OR \ L_1 \ s$

[n\_PAND\_GTO]

$\vdash \forall L. \ rv\_gt0 \ L \Rightarrow \forall s. \ 0 \leq n\_PAND \ L \ s$

[n\_PAND\_left\_flattening1]

$\vdash \forall L_2.$   
 $rv\_gt0 \ L_2 \Rightarrow$   
 $\forall L_1.$   
 $rv\_gt0 \ L_1 \Rightarrow$   
 $(n\_PAND \ (n\_PAND \ L_2 :: L_1) = n\_PAND \ (L_2 \ ++ \ L_1))$

[NOT\_MAP\_MEM]

$\vdash \forall x \ f \ L.$   
 $ALL\_DISTINCT \ L \wedge \neg MEM \ x \ L \Rightarrow \neg MEM \ \{x\} \ (MAP \ (\lambda a. \ \{a\}) \ L)$

[P\_AND\_ALWAYS]

$\vdash \forall X. \ (\forall s. \ 0 \leq X \ s) \Rightarrow (P\_AND \ ALWAYS \ X = X)$

[P\_AND\_FOLDL\_GTO]

$\vdash \forall s \ L.$   
 $rv\_gt0 \ L \Rightarrow$   
 $\forall h. \ 0 \leq h \ s \Rightarrow 0 \leq FOLDL \ (\lambda a \ b. \ P\_AND \ a \ b) \ h \ L \ s$

[P\_AND\_FOLDL\_GT01]

$\vdash \forall s \ L.$   
 $(\forall x. \ MEM \ x \ L \Rightarrow \forall s. \ 0 < x \ s) \Rightarrow$   
 $\forall h. \ 0 < h \ s \Rightarrow 0 < FOLDL \ (\lambda a \ b. \ P\_AND \ a \ b) \ h \ L \ s$

[P\_AND\_FOLDL\_NEVER]

$\vdash \forall L. \ FOLDL \ (\lambda a \ b. \ P\_AND \ a \ b) \ NEVER \ L = NEVER$

[P\_AND\_ID]

$\vdash \forall x. \ P\_AND \ x \ x = x$

[P\_AND\_lem14]

$\vdash \forall L.$   
 $L \neq [] \wedge (\forall x. \ MEM \ x \ L \Rightarrow \forall s. \ 0 < x \ s) \Rightarrow$   
 $(n\_PAND \ (L \ ++ \ [ALWAYS]) = NEVER)$

[P\_AND\_lem3]

$\vdash \forall X \ Y \ Z. \ P\_AND \ (D\_OR \ X \ Y) \ Z = D\_OR \ (P\_AND \ X \ Z) \ (P\_AND \ Y \ Z)$

[P\_AND\_NEVER]

$\vdash \forall X. \ P\_AND \ NEVER \ X = NEVER$

[PERM\_k\_out]

$\vdash \forall L_1 \ L_2 \ k. \ PERM \ L_1 \ L_2 \Rightarrow (k\_out \ k \ L_1 = k\_out \ k \ L_2)$

[PERM\_k\_out\_set\_L]

$\vdash \forall L.$   
ALL\_DISTINCT  $L \Rightarrow$   
 $PERM \ (SET\_TO\_LIST \ (k\_out \ 1 \ L)) \ (MAP \ (\lambda a. \ \{a\}) \ L)$

[PERM\_MAP\_AND\_L]

$\vdash \forall L.$   
 $rv\_gt0 \ L \Rightarrow$   
 $PERM \ (MAP \ (\lambda a. \ n\_AND \ (SET\_TO\_LIST \ a)) \ (MAP \ (\lambda a. \ \{a\}) \ L)) \ L$

[rv\_gt0\_PERM]

$\vdash \forall L_1 \ L_2. \ PERM \ L_1 \ L_2 \wedge rv\_gt0 \ L_1 \Rightarrow rv\_gt0 \ L_2$

[rv\_gt0\_PERM1]

$\vdash \forall L_1 \ L_2. \ PERM \ L_1 \ L_2 \Rightarrow rv\_gt0 \ L_1 \Rightarrow rv\_gt0 \ L_2$

[rv\_gt0\_SPLIT]

$\vdash \forall L_1 \ L_2. \ rv\_gt0 \ (L_1 \ ++ \ L_2) \iff rv\_gt0 \ L_1 \wedge rv\_gt0 \ L_2$

[SING\_1\_out\_n]

$\vdash \forall L \ s. \ s \in k\_out \ 1 \ L \Rightarrow \exists x. \ s = \{x\}$

