

Contents

1 AND_FDEP Theory

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Parent Theories: WCSP_shared

1.1 Definitions

[indep_vars3_def]

```
|- ∀X Y Z p.  
  indep_vars3 X Y Z p ⇔  
  indep_vars p (λi. lborel)  
  (λi.  
    if i = 0 then (λs. real (X s))  
    else if i = 1 then (λs. real (Y s))  
    else (λs. real (Z s))) {0; 1; 2}
```

[UNIONL_def]

```
|- (UNIONL [] = {}) ∧ ∀s ss. UNIONL (s::ss) = s ∪ UNIONL ss
```

1.2 Theorems

[AND_FDEP_event]

```
|- ∀X Y Z t p.  
  DFT_event p (D_AND X (FDEP Y Z)) t =  
  DFT_event p (D_AND X Y) t ∪ DFT_event p (D_AND X Z) t
```

[AND_FDEP_reduced]

```
|- ∀X Y Z. D_AND X (FDEP Y Z) = D_OR (D_AND X Y) (D_AND X Z)
```

[IN_MEASURABLE_borel_MAX]

```
|- ∀a f g.  
  (∀s.  
    f s ≠ PosInf ∧ f s ≠ NegInf ∧ g s ≠ PosInf ∧  
    g s ≠ NegInf) ∧ sigma_algebra a ∧  
    (λs. real (f s)) ∈ measurable a borel ∧  
    (λs. real (g s)) ∈ measurable a borel ⇒  
    (λx. real (max (f x) (g x))) ∈ measurable a borel
```

[IN_REST]

```
|- ∀x s. x ∈ REST s ⇔ x ∈ s ∧ x ≠ CHOICE s
```

[IN_UNIONL]

```
|- ∀l v. v ∈ UNIONL l ⇔ ∃s. MEM s l ∧ v ∈ s
```

[indep_vars_var]

$$\vdash \forall X \ p \ M \ s \ t \ ii. \\ s \in ii \wedge t \in ii \wedge s \neq t \Rightarrow \\ \text{indep_vars } p \ (\lambda i. \ M) \ X \ ii \Rightarrow \\ \text{indep_var } p \ M \ (X \ s) \ M \ (X \ t)$$

[lem1]

$$\vdash \forall k \ x \ y \ z \ d. \\ k \neq \text{PosInf} \wedge k \neq \text{NegInf} \wedge x \neq \text{PosInf} \wedge x \neq \text{NegInf} \wedge \\ y \neq \text{PosInf} \wedge y \neq \text{NegInf} \wedge z \neq \text{PosInf} \wedge z \neq \text{NegInf} \wedge \\ d \neq \text{PosInf} \wedge d \neq \text{NegInf} \Rightarrow \\ (k + x + -(y \times z \times d)) = k + x - y \times z \times d$$

[prob_AND_FDEP_event]

$$\vdash \forall X \ Y \ Z \ p \ t. \\ \text{rv_gt0_ninfy } [X; \ Y; \ Z] \wedge \\ \text{All_distinct_events } p \ [\text{D_AND } X \ Y; \ \text{D_AND } X \ Z] \ t \wedge \\ \text{indep_vars3 } X \ Y \ Z \ p \Rightarrow \\ (\text{prob } p \ (\text{DFT_event } p \ (\text{D_AND } X \ (\text{FDEP } Y \ Z)) \ t) = \\ \text{CDF } p \ (\lambda s. \ \text{real } (X \ s)) \ t \times \text{CDF } p \ (\lambda s. \ \text{real } (Y \ s)) \ t + \\ \text{CDF } p \ (\lambda s. \ \text{real } (X \ s)) \ t \times \text{CDF } p \ (\lambda s. \ \text{real } (Z \ s)) \ t - \\ \text{CDF } p \ (\lambda s. \ \text{real } (X \ s)) \ t \times \text{CDF } p \ (\lambda s. \ \text{real } (Y \ s)) \ t \times \\ \text{CDF } p \ (\lambda s. \ \text{real } (Z \ s)) \ t)$$

[prob_prod_3_of_3]

$$\vdash \forall p \ M \ X \ ii \ A \ s_1 \ s_2 \ s_3. \\ \text{ALL_DISTINCT } [s_1; \ s_2; \ s_3] \wedge \text{prob_space } p \wedge \\ \text{indep_vars } p \ M \ X \ ii \wedge \{s_1; \ s_2; \ s_3\} \subseteq ii \wedge \\ (\forall i. \ i \in \{s_1; \ s_2; \ s_3\} \Rightarrow A \ i \in \text{measurable_sets } (M \ i)) \Rightarrow \\ (\text{prob } p \\ (\text{PREIMAGE } (X \ s_1) \ (A \ s_1) \cap \text{p_space } p \cap \\ (\text{PREIMAGE } (X \ s_2) \ (A \ s_2) \cap \text{p_space } p) \cap \\ (\text{PREIMAGE } (X \ s_3) \ (A \ s_3) \cap \text{p_space } p)) = \\ \text{prob } p \ (\text{PREIMAGE } (X \ s_1) \ (A \ s_1) \cap \text{p_space } p) \times \\ \text{prob } p \ (\text{PREIMAGE } (X \ s_2) \ (A \ s_2) \cap \text{p_space } p) \times \\ \text{prob } p \ (\text{PREIMAGE } (X \ s_3) \ (A \ s_3) \cap \text{p_space } p))$$

[PRODUCT_UNION_3]

$$\vdash \forall s_1 \ s_2 \ s_3 \ f. \\ \text{ALL_DISTINCT } [s_1; \ s_2; \ s_3] \Rightarrow \\ (\text{product } \{s_1; \ s_2; \ s_3\} \ f = f \ s_1 \times f \ s_2 \times f \ s_3)$$