1 SEN Theory

Built: 04 November 2019
Parent Theories: DFT_DRBD

1.1 Definitions

[biginter_def]

$\vdash \forall Y\ s.$ biginter $Y\ s$ = BIGINTER $\{Y\ i \mid i \in s\}$

[bigunion_def]

$\vdash \forall Y\ s.$ bigunion $Y\ s$ = BIGUNION $\{Y\ i \mid i \in s\}$

[DISJOINT3_def]

$\vdash \forall L_1\ L_2\ L_3.$
DISJOINT3 $L_1\ L_2\ L_3 \iff$
DISJOINT $L_1\ L_2 \land$ DISJOINT $L_1\ L_3 \land$ DISJOINT $L_2\ L_3$

[event_set1_def]

$\vdash \forall X\ i\ Y.$ event_set1 $(X,i)\ Y$ = $(\lambda j.$ **if** $j = i$ **then** X **else** $Y\ j)$

[event_set2_def]

$\vdash \forall X_1\ i_1\ X_2\ i_2\ Y.$
event_set2 $(X_1,i_1)\ (X_2,i_2)\ Y$ =
event_set1 $X_1\ i_1$ (event_set1 $X_2\ i_2\ Y$)

[ind_set_def]

$\vdash \forall A.$ ind_set A = $(\lambda i.$ EL $i\ A)$

[rv_to_devent_def]

$\vdash \forall p\ X\ t.$ rv_to_devent $p\ X\ t$ = $(\lambda i.$ DFT_event $p\ (X\ i)\ t)$

[SEN_broad_set_req_def]

$\vdash \forall p\ L_1\ L_2\ L_3\ L\ A\ J\ X.$
SEN_broad_set_req $p\ L_1\ L_2\ L_3\ L\ A\ J\ X \iff$
$L_1 \neq \{\,\} \land L_2 \neq \{\,\} \land$ FINITE $L_1 \land$ FINITE $L_2 \land L_3 \neq \{\,\} \land$
FINITE $L_3 \land$
indep_sets $p\ (\lambda i.\ \{X\ i\})$ (BIGUNION_o_BIGUNION $L\ A\ J$) $\land$
disjoint_family_on (ind_set $[\{0\};\ L_1;\ L_2;\ L_3]$)
$\{0;\ 1;\ 2;\ 3\}$

[SEN_network_set_req_def]

$\vdash \forall p\ L_1\ L_2\ L_3\ L_4\ L\ LL\ A\ J\ X.$
SEN_network_set_req $p\ L_1\ L_2\ L_3\ L_4\ L\ LL\ A\ J\ X \iff$
FINITE $L_1 \wedge L_1 \neq \{\} \wedge$ FINITE $L_2 \wedge L_2 \neq \{\} \wedge$ FINITE $L_3 \wedge$
$L_3 \neq \{\} \wedge$ FINITE $L_4 \wedge L_4 \neq \{\} \wedge$ FINITE $L \wedge$
DISJOINT $\{0;\ 1;\ 3;\ 4\}\ L \wedge$
$(\forall i\ j.$
$i \in L \wedge j \in L \wedge i \neq j \Rightarrow$
DISJOINT $\{2 \times i;\ 2 \times i + 1\}\ \{2 \times j;\ 2 \times j + 1\}) \wedge$
$(\forall i.\ i \in L \Rightarrow$ DISJOINT $\{2 \times i;\ 2 \times i + 1\}\ \{0;\ 1;\ 2;\ 3;\ 4\}) \wedge$
indep_sets $p\ (\lambda i.\ \{X\ i\})$ (BIGUNION_o_BIGUNION $LL\ A\ J) \wedge$
disjoint_family_on
(ind_set
$[\{0\};\ L_1;\ L_2;\ L_3;\ L_4;$
$\{2 \times i \mid i \in L\} \cup \{2 \times i + 1 \mid i \in L\}])$
$\{0;\ 1;\ 2;\ 3;\ 4;\ 5\}$

[SEN_set_req_def]

$\vdash \forall p\ L_1\ L_2\ L\ A\ J\ X.$
SEN_set_req $p\ L_1\ L_2\ L\ A\ J\ X \iff$
$L_1 \neq \{\} \wedge L_2 \neq \{\} \wedge$ FINITE $L_1 \wedge$ FINITE $L_2 \wedge$
$(\forall l.\ l \in$ BIGUNION_o_BIGUNION $L\ A\ J \Rightarrow X\ l \in$ events $p) \wedge$
indep_sets $p\ (\lambda i.\ \{X\ i\})$ (BIGUNION_o_BIGUNION $L\ A\ J) \wedge$
disjoint_family_on (ind_set $[\{0\};\ L_1;\ L_2;\ \{3\}])$
$\{0;\ 1;\ 2;\ 3\}$

[UNIONL_def]

$\vdash$ (UNIONL [] = $\{\}) \wedge \forall s\ ss.$ UNIONL $(s::ss) = s \cup$ UNIONL ss

1.2 Theorems

[BIGINTER_4_sets]

$\vdash \forall a\ b\ c\ d.$ BIGINTER $\{a;\ b;\ c;\ d\} = a \cap b \cap c \cap d$

[BIGUNION_3_sets]

$\vdash \forall x\ y\ z.$ BIGUNION $\{x;\ y;\ z\} = x \cup y \cup z$

[BIGUNION_4_sets]

$\vdash \forall a\ b\ c\ d.$ BIGUNION $\{a;\ b;\ c;\ d\} = a \cup b \cup c \cup d$

[bigunion_biginter_bigunion_lem]

$\vdash \forall X\ s\ L\ A\ j.$
biginter
$(\lambda j.$
bigunion
$(\lambda a.$ biginter $(\lambda l.$ bigunion $X\ (s\ l))\ (L\ a))$
$(A\ j))\ \{j\} =$
bigunion $(\lambda a.$ biginter $(\lambda l.$ bigunion $X\ (s\ l))\ (L\ a))$
$(A\ j)$

[DISJOINT_def_2]

$\vdash \forall s\ t.$ DISJOINT $s\ t \iff \forall a.\ a \in s \Rightarrow a \notin t$

[DISJOINT_NUM]

$\vdash \forall a\ b\ c\ d.$
$a \neq c \land a \neq d \land b \neq c \land b \neq d \Rightarrow$ DISJOINT $\{a;\ b\}\ \{c;\ d\}$

[DRBD_parallel_bigunion]

$\vdash \forall Y\ s.$ DRBD_parallel $Y\ s$ = bigunion $Y\ s$

[DRBD_series_biginter]

$\vdash \forall Y\ s.$ DRBD_series $Y\ s$ = biginter $Y\ s$

[dSEN_n_OR]

```
⊢ ∀p Y Ys_a Ys_d Z Zs_a Zs_d X L1 L2 t.
      FINITE L1 ∧ FINITE L2 ∧
      disjoint_family_on (ind_set [{0}; L1; L2; {3}])
        {0; 1; 2; 3} ⇒
      (DFT_event p
         (n_OR
            (MAP
               (λi.
                    if i = 0 then WSP Y Ys_a Ys_d
                    else if i = 1 then
                      D_AND (n_OR (MAP X (SET_TO_LIST L1)))
                        (n_OR (MAP X (SET_TO_LIST L2)))
                    else WSP Z Zs_a Zs_d)
               (SET_TO_LIST {0; 1; 2}))) t =
       bigunion
         (λj.
              biginter
                (λa.
                     bigunion
                       (λi.
                            event_set
                              [(DFT_event p (WSP Y Ys_a Ys_d) t,
                                0);
                               (DFT_event p (WSP Z Zs_a Zs_d) t,
                                3)] (rv_to_devent p X t) i)
                       (ind_set [{0}; L1; L2; {3}] a))
                (ind_set [{0}; {1; 2}; {3}] j)) {0; 1; 2})
```

[dSEN_n_OR_BIGUNION]

```
⊢ ∀p Y Ys_a Ys_d Z Zs_a Zs_d X L1 L2 t.
      FINITE L1 ∧ FINITE L2 ∧
      disjoint_family_on (ind_set [{0}; L1; L2; {3}])
        {0; 1; 2; 3} ⇒
```

```
(DFT_event p
   (n_OR
      (MAP
         (λ i.
              if i = 0 then WSP Y Ys_a Ys_d
              else if i = 1 then
                D_AND (n_OR (MAP X (SET_TO_LIST L1)))
                  (n_OR (MAP X (SET_TO_LIST L2)))
              else WSP Z Zs_a Zs_d)
         (SET_TO_LIST {0; 1; 2}))) t =
 BIGUNION
   {BIGINTER
     {BIGUNION
       {event_set
         [(DFT_event p (WSP Y Ys_a Ys_d) t,0);
          (DFT_event p (WSP Z Zs_a Zs_d) t,3)]
         (rv_to_devent p X t) i |
       i ∈ ind_set [{0}; L1; L2; {3}] a} |
     a ∈ ind_set [{0}; {1; 2}; {3}] j} |
   j ∈ {0; 1; 2}})
```

[event_set_def]

```
⊢ (∀ i1 Y X1. event_set [(X1,i1)] Y = event_set1 (X1,i1) Y) ∧
  ∀ v8 v7 i1 Y X1.
      event_set ((X1,i1)::v7::v8) Y =
      event_set1 (X1,i1) (event_set (v7::v8) Y)
```

[event_set_ind]

```
⊢ ∀ P.
      (∀ X1 i1 Y. P [(X1,i1)] Y) ∧
      (∀ X1 i1 v7 v8 Y. P (v7::v8) Y ⇒ P ((X1,i1)::v7::v8) Y) ∧
      (∀ v4. P [] v4) ⇒
      ∀ v v1. P v v1
```

[extreal_sub_sub2]

```
⊢ ∀ a b.
      a ≠ PosInf ∧ a ≠ NegInf ∧ b ≠ PosInf ∧ b ≠ NegInf ⇒
      (a - (a - b) = b)
```

[FINITE_3]

```
⊢ ∀ a b c. FINITE {a; b; c}
```

[FINITE_4]

```
⊢ ∀ a b c d. FINITE {a; b; c; d}
```

[FINITE_PAIR]

```
⊢ ∀ s t. FINITE {s; t}
```

[IMAGE_EQ]

$\vdash \forall X\ Y\ J.\ (\forall i.\ i \in J \Rightarrow (Y\ i = X\ i)) \Rightarrow (\texttt{IMAGE}\ Y\ J = \texttt{IMAGE}\ X\ J)$

[IMAGE_EQ2]

$\vdash \forall Y\ X\ A\ J.$
$(\forall a.\ a \in \texttt{BIGUNION}\ \{A\ j \mid j \in J\} \Rightarrow (Y\ a = X\ a)) \Rightarrow$
$(\texttt{IMAGE}\ (\lambda j.\ \texttt{BIGINTER}\ (\texttt{IMAGE}\ (\lambda i.\ X\ i)\ (A\ j)))\ J =$
$\texttt{IMAGE}\ (\lambda j.\ \texttt{BIGINTER}\ (\texttt{IMAGE}\ (\lambda i.\ Y\ i)\ (A\ j)))\ J)$

[IN_REST]

$\vdash \forall x\ s.\ x \in \texttt{REST}\ s \iff x \in s \wedge x \neq \texttt{CHOICE}\ s$

[IN_UNIONL]

$\vdash \forall l\ v.\ v \in \texttt{UNIONL}\ l \iff \exists s.\ \texttt{MEM}\ s\ l \wedge v \in s$

[n_AND_BIGINTER]

$\vdash \forall p\ X\ t\ s.$
$\texttt{FINITE}\ s \wedge s \neq \{\} \wedge 0 \le t \Rightarrow$
$(\texttt{DFT\_event}\ p\ (\texttt{n\_AND}\ (\texttt{MAP}\ X\ (\texttt{SET\_TO\_LIST}\ s)))\ t =$
$\texttt{BIGINTER}\ \{\texttt{rv\_to\_devent}\ p\ X\ t\ i \mid i \in s\})$

[n_AND_BIGINTER_lem]

$\vdash \forall p\ X\ t\ s.$
$\texttt{FINITE}\ s \Rightarrow$
$0 \le t \Rightarrow$
$(\texttt{DFT\_event}\ p\ (\texttt{n\_AND}\ (\texttt{MAP}\ X\ (\texttt{SET\_TO\_LIST}\ s)))\ t =$
$\texttt{BIGINTER}\ \{\texttt{rv\_to\_devent}\ p\ X\ t\ i \mid i \in s\} \cap \texttt{p\_space}\ p)$

[n_AND_n_OR_BIGINTER_BIGUNION]

$\vdash \forall p\ X\ t\ J\ s.$
$(\forall j.\ j \in J \Rightarrow \texttt{FINITE}\ (s\ j)) \wedge \texttt{FINITE}\ J \wedge 0 \le t \wedge J \neq \{\} \Rightarrow$
$(\texttt{DFT\_event}\ p$
$(\texttt{n\_AND}$
$(\texttt{MAP}\ (\lambda j.\ \texttt{n\_OR}\ (\texttt{MAP}\ X\ (\texttt{SET\_TO\_LIST}\ (s\ j))))$
$(\texttt{SET\_TO\_LIST}\ J)))\ t =$
$\texttt{BIGINTER}$
$\{\texttt{BIGUNION}\ \{\texttt{rv\_to\_devent}\ p\ X\ t\ i \mid i \in s\ j\} \mid j \in J\})$

[n_AND_n_OR_BIGINTER_lem1]

$\vdash \forall p\ X\ t\ ii\ s.$
$\texttt{FINITE}\ s \wedge 0 \le t \wedge s \neq \{\} \Rightarrow$
$(\texttt{DFT\_event}\ p$
$(\texttt{n\_AND}$
$(\texttt{MAP}\ (\lambda i.\ \texttt{n\_OR}\ (\texttt{MAP}\ X\ (\texttt{SET\_TO\_LIST}\ (ii\ i))))$
$(\texttt{SET\_TO\_LIST}\ s)))\ t =$
$\texttt{BIGINTER}$
$\{\texttt{rv\_to\_devent}\ p$
$(\lambda i.\ \texttt{n\_OR}\ (\texttt{MAP}\ X\ (\texttt{SET\_TO\_LIST}\ (ii\ i))))\ t\ i \mid$
$i \in s\})$

[n_OR_AND_OR_BIGUNION_INTER_UNION]

```
⊢ ∀p X t L A J.
      FINITE J ∧ (∀i. i ∈ J ⇒ FINITE (A i) ∧ A i ≠ {}) ∧
      (∀i. i ∈ BIGUNION {A j | j ∈ J} ⇒ FINITE (L i)) ∧ 0 ≤ t ⇒
      (DFT_event p
         (n_OR
            (MAP
               (λj.
                    n_AND
                      (MAP
                         (λi. n_OR (MAP X (SET_TO_LIST (L i))))
                         (SET_TO_LIST (A j)))) (SET_TO_LIST J))) 
         t =
       BIGUNION
         {BIGINTER
            {BIGUNION {rv_to_devent p X t i | i ∈ L a} | a ∈ A j} |
         j ∈ J})
```

[n_OR_BIGUNION]

```
⊢ ∀p X t s.
      FINITE s ⇒
      (DFT_event p (n_OR (MAP X (SET_TO_LIST s))) t =
       BIGUNION {rv_to_devent p X t i | i ∈ s})
```

[n_OR_n_AND_BIGUNION_BIGINTER]

```
⊢ ∀p X t J s.
      (∀j. j ∈ J ⇒ FINITE (s j) ∧ s j ≠ {}) ∧ FINITE J ∧ 0 ≤ t ⇒
      (DFT_event p
         (n_OR
            (MAP (λj. n_AND (MAP X (SET_TO_LIST (s j))))
               (SET_TO_LIST J))) t =
       BIGUNION
         {BIGINTER {rv_to_devent p X t i | i ∈ s j} | j ∈ J})
```

[normal_real_mul1]

```
⊢ ∀a b c.
      a ≠ PosInf ∧ a ≠ NegInf ∧ b ≠ PosInf ∧ b ≠ NegInf ⇒
      (Normal (real a × real b × c) = a × b × Normal c)
```

[normal_real_mul2]

```
⊢ ∀a b c d.
      a ≠ PosInf ∧ a ≠ NegInf ∧ b ≠ PosInf ∧ b ≠ NegInf ⇒
      (Normal (real a × c × d × real b) =
       a × b × Normal (c × d))
```

[normal_real_mul3]

```
⊢ ∀a b c.
      a ≠ PosInf ∧ a ≠ NegInf ∧ b ≠ PosInf ∧ b ≠ NegInf ⇒
      (Normal (real a × c × real b) = a × b × Normal c)
```

[PROB_bigunion_biginter_bigunion_lem1]

```
⊢ ∀p X s L A j.
      indep_sets p (λi. {X i}) (BIGUNION_o_BIGUNION s L (A j)) ∧
      sets_finite_not_empty s L A {j} ⇒
      (prob p
         (biginter
            (λj.
                 bigunion
                   (λa. biginter (λl. bigunion X (s l)) (L a))
                   (A j)) {j}) =
       1 -
       Normal
         (product (A j)
            (λa.
                 real
                   (1 -
                    Normal
                      (product (L a)
                         (λl.
                              real
                                (1 -
                                 Normal
                                   (product (s l)
                                      (λi.
                                           real
                                             (1 - prob p (X i))))))))))
```

[PROB_bigunion_biginter_bigunion_lem2]

```
⊢ ∀p X s L A j.
      indep_sets p (λi. {X i}) (BIGUNION_o_BIGUNION s L (A j)) ∧
      sets_finite_not_empty s L A {j} ⇒
      (prob p
         (bigunion (λa. biginter (λl. bigunion X (s l)) (L a))
            (A j)) =
       1 -
       Normal
         (product (A j)
            (λa.
                 real
                   (1 -
                    Normal
                      (product (L a)
                         (λl.
                              real
                                (1 -
                                 Normal
                                   (product (s l)
                                      (λi.
```

```
                                          real
                                            (1 - prob p (X i))))))))))
```

[PROB_DFT_SEN_plus]

```
⊢ ∀p X Y Ys_a Ys_d Z Zs_a Zs_d t L₁ L₂.
      0 ≤ t ∧
      SEN_set_req p L₁ L₂ (ind_set [{0}; L₁; L₂; {3}])
        (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
        (λi.
             event_set
               [(DFT_event p (WSP Y Ys_a Ys_d) t,0);
                (DFT_event p (WSP Z Zs_a Zs_d) t,3)]
               (rv_to_devent p X t) i) ∧
      (∀i. i ∈ L₁ ∪ L₂ ⇒ rv_gt0_ninfinity [X i]) ⇒
      (prob p
         (DFT_event p
            (n_OR
               (MAP
                  (λi.
                       if i = 0 then WSP Y Ys_a Ys_d
                       else if i = 1 then
                         D_AND (n_OR (MAP X (SET_TO_LIST L₁)))
                           (n_OR (MAP X (SET_TO_LIST L₂)))
                       else WSP Z Zs_a Zs_d)
                  (SET_TO_LIST {0; 1; 2}))) t) =
       1 -
       (1 - prob p (DFT_event p (WSP Y Ys_a Ys_d) t)) ×
       (Normal
          (1 -
           (1 -
            product L₁ (λi. real (1 - CDF p (real ∘ X i) t))) ×
           (1 -
            product L₂ (λi. real (1 - CDF p (real ∘ X i) t)))) ×
        (1 - prob p (DFT_event p (WSP Z Zs_a Zs_d) t))))
```

[PROB_DFT_SEN_plus_broadcast]

```
⊢ ∀p X Y Ysa Ysd t L₁ L₂ L₃.
      0 ≤ t ∧ (∀i. i ∈ L₁ ∪ L₂ ∪ L₃ ⇒ rv_gt0_ninfinity [X i]) ∧
      SEN_broad_set_req p L₁ L₂ L₃ (ind_set [{0}; L₁; L₂; L₃])
        (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
        (event_set [(DFT_event p (WSP Y Ysa Ysd) t,0)]
           (rv_to_devent p X t)) ⇒
      (prob p
         (DFT_event p
            (n_OR
               (MAP
                  (λj.
                       n_AND
                         (MAP
```

```
                              (λ i.
                                   n_OR
                                     (MAP
                                        (λ i.
                                            if i = 0 then
                                              WSP Y Ysa Ysd
                                            else X i)
                                        (SET_TO_LIST
                                           (ind_set
                                              [{0}; L₁; L₂; L₃] i))))
                              (SET_TO_LIST
                                 (ind_set [{0}; {1; 2}; {3}] j))))
                     (SET_TO_LIST {0; 1; 2}))) t) =
          1 -
          (1 - prob p (DFT_event p (WSP Y Ysa Ysd) t)) ×
          (Normal
             (1 -
              (1 -
               product L₁ (λ i. real (1 - CDF p (real ∘ X i) t))) ×
              (1 -
               product L₂ (λ i. real (1 - CDF p (real ∘ X i) t)))) ×
           Normal
             (product L₃ (λ i. real (1 - CDF p (real ∘ X i) t))))
```

[PROB_DFT_SEN_plus_broadcast_final]

```
 ⊢ ∀ p X Y Ysa Ysd t L₁ L₂ L₃.
        SEN_broad_set_req p L₁ L₂ L₃ (ind_set [{0}; L₁; L₂; L₃])
          (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
          (event_set [(DFT_event p (WSP Y Ysa Ysd) t,0)]
             (rv_to_devent p X t)) ∧ 0 ≤ t ∧
        (∀ i. i ∈ L₁ ∪ L₂ ∪ L₃ ⇒ rv_gt0_ninfinity [X i]) ⇒
        (prob p
           (DFT_event p
              (n_OR
                 (MAP
                    (λ i.
                         if i = 0 then WSP Y Ysa Ysd
                         else if i = 1 then
                           D_AND (n_OR (MAP X (SET_TO_LIST L₁)))
                             (n_OR (MAP X (SET_TO_LIST L₂)))
                         else n_OR (MAP X (SET_TO_LIST L₃)))
                    (SET_TO_LIST {0; 1; 2}))) t) =
          1 -
          (1 - prob p (DFT_event p (WSP Y Ysa Ysd) t)) ×
          (Normal
             (1 -
              (1 -
               product L₁ (λ i. real (1 - CDF p (real ∘ X i) t))) ×
              (1 -
```

```
      product L2 (λi. real (1 - CDF p (real ∘ X i) t)))) ×
    Normal
      (product L3 (λi. real (1 - CDF p (real ∘ X i) t)))))
```

[PROB_DFT_SEN_plus_broadcast_lem1]

```
⊢ ∀p X L1 L2 L3 s L A j.
      L1 ≠ {} ∧ L2 ≠ {} ∧ L3 ≠ {} ∧ FINITE L1 ∧ FINITE L2 ∧
      FINITE L3 ∧
      indep_sets p (λi. {X i}) (BIGUNION_o_BIGUNION s L A) ∧
      disjoint_family_on (ind_set [{0}; L1; L2; L3])
        {0; 1; 2; 3} ∧ (A = {0; 1; 2}) ∧
      (L = ind_set [{0}; {1; 2}; {3}]) ∧
      (s = ind_set [{0}; L1; L2; L3]) ⇒
      (prob p
         (bigunion (λa. biginter (λl. bigunion X (s l)) (L a))
            ((λi. A) j)) =
       1 -
       (1 - prob p (X 0)) ×
       (Normal
          (1 -
           (1 - product L1 (λi. real (1 - prob p (X i)))) ×
           (1 - product L2 (λi. real (1 - prob p (X i))))) ×
        Normal (product L3 (λi. real (1 - prob p (X i))))))
```

[PROB_DFT_SEN_plus_broadcast_lem2]

```
⊢ ∀p X Y Ys_a Ys_d t L1 L2 L3.
      0 ≤ t ∧ FINITE L1 ∧ FINITE L2 ∧ FINITE L3 ∧ L1 ≠ {} ∧
      L2 ≠ {} ∧ L3 ≠ {} ∧
      (∀i. i ∈ L1 ∪ L2 ∪ L3 ⇒ rv_gt0_ninfinity [X i]) ∧
      indep_sets p (λi. {rv_to_devent p X t i})
        (BIGUNION_o_BIGUNION (ind_set [{0}; L1; L2; L3])
           (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}) ∧
      disjoint_family_on (ind_set [{0}; L1; L2; L3])
        {0; 1; 2; 3} ⇒
      (prob p
         (DFT_event p
            (n_OR
               (MAP
                  (λj.
                       n_AND
                         (MAP
                            (λi.
                                 n_OR
                                   (MAP X
                                      (SET_TO_LIST
                                         (ind_set
                                            [{0}; L1; L2; L3] i))))
                            (SET_TO_LIST
                               (ind_set [{0}; {1; 2}; {3}] j))))
```

```
              (SET_TO_LIST {0; 1; 2}))) t) =
   1 -
   (1 - prob p (rv_to_devent p X t 0)) ×
   (Normal
      (1 -
       (1 -
        product L1 (λi. real (1 - CDF p (real ∘ X i) t))) ×
       (1 -
        product L2 (λi. real (1 - CDF p (real ∘ X i) t)))) ×
    Normal
      (product L3 (λi. real (1 - CDF p (real ∘ X i) t))))
```

[PROB_DFT_SEN_plus_lem1]

```
⊢ ∀p X L1 L2 s L A j.
     L1 ≠ { } ∧ L2 ≠ { } ∧ FINITE L1 ∧ FINITE L2 ∧
     indep_sets p (λi. {X i}) (BIGUNION_o_BIGUNION s L A) ∧
     disjoint_family_on (ind_set [{0}; L1; L2; {3}])
       {0; 1; 2; 3} ∧ (A = {0; 1; 2}) ∧
     (L = ind_set [{0}; {1; 2}; {3}]) ∧
     (s = ind_set [{0}; L1; L2; {3}]) ⇒
     (prob p
        (bigunion (λa. biginter (λl. bigunion X (s l)) (L a))
           ((λi. A) j)) =
      1 -
      (1 - prob p (X 0)) ×
      (Normal
         (1 -
          (1 - product L1 (λi. real (1 - prob p (X i)))) ×
          (1 - product L2 (λi. real (1 - prob p (X i))))) ×
       (1 - prob p (X 3))))
```

[PROB_DFT_SEN_plus_lem2]

```
⊢ ∀p X Y Ys_a Ys_d Z Zs_a Zs_d t L1 L2.
     0 ≤ t ∧ FINITE L1 ∧ FINITE L2 ∧ L1 ≠ { } ∧ L2 ≠ { } ∧
     indep_sets p (λi. {rv_to_devent p X t i})
       (BIGUNION_o_BIGUNION (ind_set [{0}; L1; L2; {3}])
          (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}) ∧
     disjoint_family_on (ind_set [{0}; L1; L2; {3}])
       {0; 1; 2; 3} ⇒
     (prob p
        (DFT_event p
           (n_OR
              (MAP
                 (λj.
                      n_AND
                        (MAP
                           (λi.
                                n_OR
                                  (MAP X
```

```
                                    (SET_TO_LIST
                                       (ind_set
                                          [{0}; L₁; L₂; {3}]
                                          i))))
                          (SET_TO_LIST
                             (ind_set [{0}; {1; 2}; {3}] j))))
                (SET_TO_LIST {0; 1; 2}))) t) =
        1 -
        (1 - prob p (rv_to_devent p X t 0)) ×
        (Normal
           (1 -
            (1 -
             product L₁
               (λ i. real (1 - prob p (rv_to_devent p X t i)))) ×
            (1 -
             product L₂
               (λ i. real (1 - prob p (rv_to_devent p X t i))))) ×
         (1 - prob p (rv_to_devent p X t 3))))
```

[PROB_DFT_SEN_plus_lem3]

```
 ⊢ ∀ p X Y Ys_a Ys_d Z Zs_a Zs_d t L₁ L₂.
       0 ≤ t ∧
       SEN_set_req p L₁ L₂ (ind_set [{0}; L₁; L₂; {3}])
         (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
         (λ i.
              event_set
                [(DFT_event p (WSP Y Ys_a Ys_d) t,0);
                 (DFT_event p (WSP Z Zs_a Zs_d) t,3)]
                (rv_to_devent p X t) i) ⇒
       (prob p
          (DFT_event p
             (n_OR
                (MAP
                   (λ i.
                        if i = 0 then WSP Y Ys_a Ys_d
                        else if i = 1 then
                          D_AND (n_OR (MAP X (SET_TO_LIST L₁)))
                            (n_OR (MAP X (SET_TO_LIST L₂)))
                        else WSP Z Zs_a Zs_d)
                   (SET_TO_LIST {0; 1; 2}))) t) =
        1 -
        (1 -
         prob p
           (rv_to_devent p
              (λ i.
                   if i = 0 then WSP Y Ys_a Ys_d
                   else if i = 3 then WSP Z Zs_a Zs_d
                   else X i) t 0)) ×
        (Normal
```

```
          (1 -
           (1 -
            product L₁
              (λ i.
                   real
                     (1 -
                      prob p
                        (rv_to_devent p
                           (λ i.
                                if i = 0 then WSP Y Ys_a Ys_d
                                else if i = 3 then
                                  WSP Z Zs_a Zs_d
                                else X i) t i)))) ×
           (1 -
            product L₂
              (λ i.
                   real
                     (1 -
                      prob p
                        (rv_to_devent p
                           (λ i.
                                if i = 0 then WSP Y Ys_a Ys_d
                                else if i = 3 then
                                  WSP Z Zs_a Zs_d
                                else X i) t i))))) ×
        (1 -
         prob p
           (rv_to_devent p
              (λ i.
                   if i = 0 then WSP Y Ys_a Ys_d
                   else if i = 3 then WSP Z Zs_a Zs_d
                   else X i) t 3))))
```

[PROB_DRBD_SEN_network]

```
 ⊢ ∀p L₁ L₂ L₃ L₄ L X Y t.
       SEN_network_set_req p L₁ L₂ L₃ L₄ L
         (λ i.
              if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
                {i}
              else ind_set [{0}; L₁; L₂; L₃; L₄] i)
         (λ j.
              if j ∈ L then {2 × j; 2 × j + 1}
              else ind_set [{0}; {1}; {}; {2; 3}; {4}] j)
         ({0; 1; 3; 4} ∪ L)
         (event_set [(DRBD_event p Y t,0)] (rv_to_event p X t)) ⇒
       (prob p
          (DRBD_event p
             (nR_AND
                (λ i.
```

```
                 if i = 0 then Y
                 else if i = 1 then nR_AND X L1
                 else if i = 3 then
                   R_OR (nR_AND X L2) (nR_AND X L3)
                 else if i = 4 then nR_AND X L4
                 else R_OR (X (2 × i)) (X (2 × i + 1)))
             ({0; 1; 3; 4} ∪ L)) t) =
      Rel p Y t ×
      Normal (product L1 (λl. real (Rel p (X l) t))) ×
      (1 -
       (1 - Normal (product L2 (λl. real (Rel p (X l) t)))) ×
       (1 - Normal (product L3 (λl. real (Rel p (X l) t))))) ×
      Normal (product L4 (λl. real (Rel p (X l) t))) ×
      Normal
        (product L
           (λj.
                1 -
                real
                  ((1 - Rel p (X (2 × j)) t) ×
                   (1 - Rel p (X (2 × j + 1)) t)))))
```

[PROB_DRBD_SEN_network_one_spare]

```
 ⊢ ∀p L1 L2 L3 L4 L X Y Ysa Ysd t.
     SEN_network_set_req p L1 L2 L3 L4 L
       (λi.
            if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
              {i}
            else ind_set [{0}; L1; L2; L3; L4] i)
       (λj.
            if j ∈ L then {2 × j; 2 × j + 1}
            else ind_set [{0}; {1}; { }; {2; 3}; {4}] j)
       ({0; 1; 3; 4} ∪ L)
       (event_set [(DRBD_event p (R_WSP Y Ysa Ysd) t,0)]
          (rv_to_event p X t)) ⇒
     (prob p
        (DRBD_event p
           (nR_AND
              (λi.
                 if i = 0 then R_WSP Y Ysa Ysd
                 else if i = 1 then nR_AND X L1
                 else if i = 3 then
                   R_OR (nR_AND X L2) (nR_AND X L3)
                 else if i = 4 then nR_AND X L4
                 else R_OR (X (2 × i)) (X (2 × i + 1)))
             ({0; 1; 3; 4} ∪ L)) t) =
      Rel p (R_WSP Y Ysa Ysd) t ×
      Normal (product L1 (λl. real (Rel p (X l) t))) ×
      (1 -
       (1 - Normal (product L2 (λl. real (Rel p (X l) t)))) ×
```

```
      (1 - Normal (product L₃ (λl. real (Rel p (X l) t))))) ×
     Normal (product L₄ (λl. real (Rel p (X l) t))) ×
     Normal
       (product L
          (λj.
               1 -
               real
                 ((1 - Rel p (X (2 × j)) t) ×
                  (1 - Rel p (X (2 × j + 1)) t)))))
```

[PROB_DRBD_SEN_network_spares]

```
 ⊢ ∀p L₁ L₂ L₃ L₄ L X Y Ysa Ysd t.
       SEN_network_set_req p L₁ L₂ L₃ L₄ L
         (λi.
              if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
                {i}
              else ind_set [{0}; L₁; L₂; L₃; L₄] i)
         (λj.
              if j ∈ L then {2 × j; 2 × j + 1}
              else ind_set [{0}; {1}; {}; {2; 3}; {4}] j)
         ({0; 1; 3; 4} ∪ L)
         (event_set
            [(DRBD_event p (R_WSP (Y 0) (Ysa 0) (Ysd 0)) t,0)]
            (rv_to_event p X t)) ∧
       (∀i. i ∈ L₁ ⇒ (X i = R_WSP (Y i) (Ysa i) (Ysd i))) ⇒
       (prob p
          (DRBD_event p
             (nR_AND
                (λi.
                     if i = 0 then R_WSP (Y 0) (Ysa 0) (Ysd 0)
                     else if i = 1 then nR_AND X L₁
                     else if i = 3 then
                       R_OR (nR_AND X L₂) (nR_AND X L₃)
                     else if i = 4 then nR_AND X L₄
                     else R_OR (X (2 × i)) (X (2 × i + 1)))
                ({0; 1; 3; 4} ∪ L)) t) =
        Normal
          (product ({0} ∪ L₁)
             (λl. real (Rel p (R_WSP (Y l) (Ysa l) (Ysd l)) t))) ×
        (1 -
         (1 - Normal (product L₂ (λl. real (Rel p (X l) t)))) ×
         (1 - Normal (product L₃ (λl. real (Rel p (X l) t))))) ×
        Normal (product L₄ (λl. real (Rel p (X l) t))) ×
        Normal
          (product L
             (λj.
                  1 -
                  real
                    ((1 - Rel p (X (2 × j)) t) ×
```

```
                    (1 - Rel p (X (2 × j + 1)) t)))))
```

[PROB_DRBD_SEN_plus]

```
 ⊢ ∀p X Y Z t L₁ L₂ L A J.
       L₁ ≠ { } ∧ L₂ ≠ { } ∧ FINITE L₁ ∧ FINITE L₂ ∧
       (∀l.
            l ∈ BIGUNION_o_BIGUNION L A J ⇒
            event_set
              [(DRBD_event p Y t,0); (DRBD_event p Z t,3)] X l ∈
            events p) ∧
       indep_sets p
         (λi.
              {event_set
                 [(DRBD_event p Y t,0); (DRBD_event p Z t,3)] X
                 i}) (BIGUNION_o_BIGUNION L A J) ∧
       disjoint_family_on (ind_set [{0}; L₁; L₂; {3}])
         {0; 1; 2; 3} ∧ (J = {0; 1; 2}) ∧
       (A = ind_set [{0}; {1; 2}; {3}]) ∧
       (L = ind_set [{0}; L₁; L₂; {3}]) ⇒
       (prob p
          (DRBD_series
             (λj.
                  DRBD_parallel
                    (λa.
                         DRBD_series
                           (λi.
                                event_set
                                  [(DRBD_event p Y t,0);
                                   (DRBD_event p Z t,3)] X i)
                           (L a)) (A j)) J) =
        prob p (DRBD_event p Y t) × prob p (DRBD_event p Z t) ×
        (1 -
         (1 - Normal (product L₁ (λl. real (prob p (X l))))) ×
         (1 - Normal (product L₂ (λl. real (prob p (X l)))))))
```

[PROB_DRBD_SEN_plus_broadcast]

```
 ⊢ ∀p X Y t L₁ L₂ L₃ L A J.
       L₁ ≠ { } ∧ L₂ ≠ { } ∧ L₃ ≠ { } ∧ FINITE L₁ ∧ FINITE L₂ ∧
       FINITE L₃ ∧
       indep_sets p (λi. {event_set [(DRBD_event p Y t,0)] X i})
         (BIGUNION_o_BIGUNION L A J) ∧
       disjoint_family_on (ind_set [{0}; L₁; L₂; L₃])
         {0; 1; 2; 3} ∧ (J = {0; 1; 2}) ∧
       (A = ind_set [{0}; {1; 2}; {3}]) ∧
       (L = ind_set [{0}; L₁; L₂; L₃]) ⇒
       (prob p
          (DRBD_series
             (λj.
                  DRBD_parallel
```

```
                   (λ a.
                        DRBD_series
                          (λ i.
                               event_set [(DRBD_event p Y t,0)]
                                 X i) (L a)) (A j)) J) =
        prob p (DRBD_event p Y t) ×
        Normal (product L3 (λ l. real (prob p (X l)))) ×
        (1 -
         (1 - Normal (product L1 (λ l. real (prob p (X l))))) ×
         (1 - Normal (product L2 (λ l. real (prob p (X l))))))
```

[PROB_DRBD_SEN_plus_broadcast_lem1]

```
 ⊢ ∀ p X L1 L2 L3 L A J.
       L1 ≠ { } ∧ L2 ≠ { } ∧ FINITE L1 ∧ FINITE L2 ∧ FINITE L3 ∧
       L3 ≠ { } ∧
       indep_sets p (λ i. {X i}) (BIGUNION_o_BIGUNION L A J) ∧
       disjoint_family_on (ind_set [{0}; L1; L2; L3])
         {0; 1; 2; 3} ∧ (J = {0; 1; 2}) ∧
       (A = ind_set [{0}; {1; 2}; {3}]) ∧
       (L = ind_set [{0}; L1; L2; L3]) ⇒
       (prob p
          (DRBD_series
             (λ j. DRBD_parallel (λ a. DRBD_series X (L a)) (A j))
             J) =
        prob p (X 0) ×
        Normal (product L3 (λ l. real (prob p (X l)))) ×
        (1 -
         (1 - Normal (product L1 (λ l. real (prob p (X l))))) ×
         (1 - Normal (product L2 (λ l. real (prob p (X l)))))))
```

[PROB_DRBD_SEN_plus_broadcast_rel_lem]

```
 ⊢ ∀ p X Y Ys_a Ys_d t L1 L2 L3 L A J.
       L1 ≠ { } ∧ L2 ≠ { } ∧ L3 ≠ { } ∧ FINITE L1 ∧ FINITE L2 ∧
       FINITE L3 ∧
       indep_sets p
         (λ i.
              {event_set
                 [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0)] X i})
         (BIGUNION_o_BIGUNION L A J) ∧
       disjoint_family_on (ind_set [{0}; L1; L2; L3])
         {0; 1; 2; 3} ∧ (J = {0; 1; 2}) ∧
       (A = ind_set [{0}; {1; 2}; {3}]) ∧
       (L = ind_set [{0}; L1; L2; L3]) ⇒
       (prob p
          (DRBD_series
             (λ j.
                  DRBD_parallel
                    (λ a.
                         DRBD_series
```

```
                         (λi.
                              event_set
                                [(DRBD_event p
                                    (R_WSP Y Ys_a Ys_d) t,0)]
                                X i) (L a)) (A j)) J) =
        Rel p (R_WSP Y Ys_a Ys_d) t ×
        Normal (product L₃ (λl. real (prob p (X l)))) ×
        (1 -
         (1 - Normal (product L₁ (λl. real (prob p (X l))))) ×
         (1 - Normal (product L₂ (λl. real (prob p (X l))))))
```

[PROB_DRBD_SEN_plus_lem1]

```
 ⊢ ∀p X L₁ L₂ L A J.
       L₁ ≠ { } ∧ L₂ ≠ { } ∧ FINITE L₁ ∧ FINITE L₂ ∧
       (∀l. l ∈ BIGUNION_o_BIGUNION L A J ⇒ X l ∈ events p) ∧
       indep_sets p (λi. {X i}) (BIGUNION_o_BIGUNION L A J) ∧
       disjoint_family_on (ind_set [{0}; L₁; L₂; {3}])
         {0; 1; 2; 3} ∧ (J = {0; 1; 2}) ∧
       (A = ind_set [{0}; {1; 2}; {3}]) ∧
       (L = ind_set [{0}; L₁; L₂; {3}]) ⇒
       (prob p
          (DRBD_series
             (λj. DRBD_parallel (λa. DRBD_series X (L a)) (A j))
             J) =
        prob p (X 0) × prob p (X 3) ×
        (1 -
         (1 - Normal (product L₁ (λl. real (prob p (X l))))) ×
         (1 - Normal (product L₂ (λl. real (prob p (X l)))))))
```

[PROB_DRBD_SEN_plus_rel]

```
 ⊢ ∀p X Y Ys_a Ys_d t L₁ L₂ L₃ L A J.
       SEN_broad_set_req p L₁ L₂ L₃ (ind_set [{0}; L₁; L₂; L₃])
         (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
         (event_set [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0)]
            (rv_to_event p X t)) ⇒
       (prob p
          (DRBD_series
             (λj.
                  DRBD_parallel
                    (λa.
                         DRBD_series
                           (λi.
                                event_set
                                  [(DRBD_event p
                                      (R_WSP Y Ys_a Ys_d) t,0)]
                                  (rv_to_event p X t) i)
                           (ind_set [{0}; L₁; L₂; L₃] a))
                    (ind_set [{0}; {1; 2}; {3}] j)) {0; 1; 2}) =
        Rel p (R_WSP Y Ys_a Ys_d) t ×
```

```
        Normal (product L₃ (λ l. real (Rel p (X l) t))) ×
        (1 -
         (1 - Normal (product L₁ (λ l. real (Rel p (X l) t)))) ×
         (1 - Normal (product L₂ (λ l. real (Rel p (X l) t))))))
```

[PROB_DRBD_SEN_plus_rel_lem]

```
⊢ ∀ p X Y Ys_a Ys_d Z Zs_a Zs_d t L₁ L₂ L A J.
       L₁ ≠ { } ∧ L₂ ≠ { } ∧ FINITE L₁ ∧ FINITE L₂ ∧
       (∀ l.
            l ∈ BIGUNION_o_BIGUNION L A J ⇒
            event_set
              [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0);
               (DRBD_event p (R_WSP Z Zs_a Zs_d) t,3)] X l ∈
            events p) ∧
       indep_sets p
         (λ i.
              {event_set
                 [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0);
                  (DRBD_event p (R_WSP Z Zs_a Zs_d) t,3)] X i})
         (BIGUNION_o_BIGUNION L A J) ∧
       disjoint_family_on (ind_set [{0}; L₁; L₂; {3}])
         {0; 1; 2; 3} ∧ (J = {0; 1; 2}) ∧
       (A = ind_set [{0}; {1; 2}; {3}]) ∧
       (L = ind_set [{0}; L₁; L₂; {3}]) ⇒
       (prob p
          (DRBD_series
             (λ j.
                  DRBD_parallel
                    (λ a.
                         DRBD_series
                           (λ i.
                                event_set
                                  [(DRBD_event p
                                      (R_WSP Y Ys_a Ys_d) t,0);
                                   (DRBD_event p
                                      (R_WSP Z Zs_a Zs_d) t,3)]
                                  X i) (L a)) (A j)) J) =
        Rel p (R_WSP Y Ys_a Ys_d) t ×
        Rel p (R_WSP Z Zs_a Zs_d) t ×
        (1 -
         (1 - Normal (product L₁ (λ l. real (prob p (X l))))) ×
         (1 - Normal (product L₂ (λ l. real (prob p (X l)))))))
```

[PROB_n_AND]

```
⊢ ∀ p X t s.
       FINITE s ∧ s ≠ { } ∧ 0 ≤ t ∧
       indep_sets p (λ i. {rv_to_devent p X t i}) s ⇒
       (prob p (DFT_event p (n_AND (MAP X (SET_TO_LIST s))) t) =
        Normal
          (product s (λ i. real (prob p (rv_to_devent p X t i)))))
```

[PROB_n_AND_CDF]

```
⊢ ∀p X t s.
      FINITE s ∧ s ≠ { } ∧ 0 ≤ t ∧
      indep_sets p (λi. {rv_to_devent p X t i}) s ∧
      (∀i. i ∈ s ⇒ rv_gt0_ninfinity [X i]) ⇒
      (prob p (DFT_event p (n_AND (MAP X (SET_TO_LIST s))) t) =
       Normal (product s (λi. real (CDF p (real ∘ X i) t))))
```

[PROB_n_OR]

```
⊢ ∀p X t s.
      indep_sets p (λi. {rv_to_devent p X t i}) s ∧ s ≠ { } ∧
      FINITE s ⇒
      (prob p (DFT_event p (n_OR (MAP X (SET_TO_LIST s))) t) =
       1 -
       Normal
         (product s
            (λi. real (1 - prob p (rv_to_devent p X t i)))))
```

[PROB_n_OR_CDF]

```
⊢ ∀p X t s.
      s ≠ { } ∧ FINITE s ∧
      indep_sets p (λi. {rv_to_devent p X t i}) s ∧
      (∀i. i ∈ s ⇒ rv_gt0_ninfinity [X i]) ⇒
      (prob p (DFT_event p (n_OR (MAP X (SET_TO_LIST s))) t) =
       1 -
       Normal (product s (λi. real (1 - CDF p (real ∘ X i) t))))
```

[PROB_SEN_network_DFT_lem1]

```
⊢ ∀p L1 L2 L3 L4 L X t LL A J.
      FINITE L1 ∧ L1 ≠ { } ∧ FINITE L2 ∧ L2 ≠ { } ∧ FINITE L3 ∧
      L3 ≠ { } ∧ FINITE L4 ∧ L4 ≠ { } ∧ FINITE L ∧
      DISJOINT {0; 1; 3; 4} L ∧
      (∀i j.
           i ∈ L ∧ j ∈ L ∧ i ≠ j ⇒
           DISJOINT {2 × i; 2 × i + 1} {2 × j; 2 × j + 1}) ∧
      (∀i. i ∈ L ⇒ DISJOINT {2 × i; 2 × i + 1} {0; 1; 2; 3; 4}) ∧
      indep_sets p (λi. {X i}) (BIGUNION_o_BIGUNION LL A J) ∧
      disjoint_family_on
        (ind_set
           [{0}; L1; L2; L3; L4;
            {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L}])
        {0; 1; 2; 3; 4; 5} ∧ (J = {0; 1; 3; 4} ∪ L) ∧
      (A =
       (λj.
            if j ∈ L then {2 × j; 2 × j + 1}
            else ind_set [{0}; {1}; { }; {2; 3}; {4}] j)) ∧
      (LL =
       (λi.
```

```
        if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
          {i}
        else ind_set [{0}; L₁; L₂; L₃; L₄] i)) ⇒
   (prob p
      (bigunion (λj. biginter (λa. bigunion X (LL a)) (A j))
         J) =
    1 -
    (1 - prob p (X 0)) ×
    Normal (product L₁ (λl. real (1 - prob p (X l)))) ×
    (1 -
     (1 - Normal (product L₂ (λl. real (1 - prob p (X l))))) ×
     (1 - Normal (product L₃ (λl. real (1 - prob p (X l)))))) ×
    Normal (product L₄ (λl. real (1 - prob p (X l)))) ×
    Normal
      (product L
         (λj.
              1 -
              real
                (prob p (X (2 × j)) × prob p (X (2 × j + 1))))))
```

[PROB_SEN_network_DFT_lem2]

```
 ⊢ ∀p L₁ L₂ L₃ L₄ L X Y t LL A J.
     FINITE L₁ ∧ L₁ ≠ {} ∧ FINITE L₂ ∧ L₂ ≠ {} ∧ FINITE L₃ ∧
     L₃ ≠ {} ∧ FINITE L₄ ∧ L₄ ≠ {} ∧ FINITE L ∧
     DISJOINT {0; 1; 3; 4} L ∧
     (∀i j.
          i ∈ L ∧ j ∈ L ∧ i ≠ j ⇒
          DISJOINT {2 × i; 2 × i + 1} {2 × j; 2 × j + 1}) ∧
     (∀i. i ∈ L ⇒ DISJOINT {2 × i; 2 × i + 1} {0; 1; 2; 3; 4}) ∧
     indep_sets p
       (λi.
           {event_set [(DFT_event p Y t,0)]
              (rv_to_devent p X t) i})
       (BIGUNION_o_BIGUNION LL A J) ∧
     disjoint_family_on
       (ind_set
          [{0}; L₁; L₂; L₃; L₄;
           {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L}])
       {0; 1; 2; 3; 4; 5} ∧ (J = {0; 1; 3; 4} ∪ L) ∧
     (A =
      (λj.
           if j ∈ L then {2 × j; 2 × j + 1}
           else ind_set [{0}; {1}; {}; {2; 3}; {4}] j)) ∧
     (LL =
      (λi.
           if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
             {i}
           else ind_set [{0}; L₁; L₂; L₃; L₄] i)) ⇒
     (prob p
```

```
    (bigunion
       (λj.
            biginter
              (λa.
                   bigunion
                     (λi.
                          event_set [(DFT_event p Y t,0)]
                            (rv_to_devent p X t) i)
                     ((λi.
                           if
                             i ∈
                             {2 × i | i ∈ L} ∪
                             {2 × i + 1 | i ∈ L}
                           then
                             {i}
                           else
                             ind_set [{0}; L1; L2; L3; L4]
                               i) a))
              ((λj.
                    if j ∈ L then {2 × j; 2 × j + 1}
                    else
                      ind_set [{0}; {1}; {}; {2; 3}; {4}]
                        j) j)) ({0; 1; 3; 4} ∪ L)) =
  1 -
  (1 - prob p (DFT_event p Y t)) ×
  Normal
    (product L1
       (λl. real (1 - prob p (rv_to_devent p X t l)))) ×
  (1 -
   (1 -
    Normal
      (product L2
         (λl. real (1 - prob p (rv_to_devent p X t l))))) ×
   (1 -
    Normal
      (product L3
         (λl. real (1 - prob p (rv_to_devent p X t l)))))) ×
  Normal
    (product L4
       (λl. real (1 - prob p (rv_to_devent p X t l)))) ×
  Normal
    (product L
       (λj.
            1 -
            real
              (prob p (rv_to_devent p X t (2 × j)) ×
               prob p (rv_to_devent p X t (2 × j + 1))))))
```

[PROB_SEN_network_DFT_lem3]

```
⊢ ∀p L1 L2 L3 L4 L X Y Ysa Ysd t.
     SEN_network_set_req p L1 L2 L3 L4 L
       (λi.
            if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
              {i}
            else ind_set [{0}; L1; L2; L3; L4] i)
       (λj.
            if j ∈ L then {2 × j; 2 × j + 1}
            else ind_set [{0}; {1}; {}; {2; 3}; {4}] j)
       ({0; 1; 3; 4} ∪ L)
       (λi.
            event_set [(DFT_event p (WSP Y Ysa Ysd) t,0)]
              (rv_to_devent p X t) i) ∧
     (∀i.
          i ∈
          L1 ∪ L2 ∪ L3 ∪ L4 ∪ {2 × i | i ∈ L} ∪
          {2 × i + 1 | i ∈ L} ⇒
          rv_gt0_ninfinity [X i]) ⇒
     (prob p
        (bigunion
           (λj.
                biginter
                  (λa.
                       bigunion
                         (λi.
                              event_set
                                [(DFT_event p (WSP Y Ysa Ysd)
                                    t,0)] (rv_to_devent p X t)
                                i)
                         ((λi.
                               if
                                 i ∈
                                 {2 × i | i ∈ L} ∪
                                 {2 × i + 1 | i ∈ L}
                               then
                                 {i}
                               else
                                 ind_set [{0}; L1; L2; L3; L4]
                                   i) a))
                  ((λj.
                        if j ∈ L then {2 × j; 2 × j + 1}
                        else
                          ind_set [{0}; {1}; {}; {2; 3}; {4}]
                            j) j)) ({0; 1; 3; 4} ∪ L)) =
      1 -
      (1 - prob p (DFT_event p (WSP Y Ysa Ysd) t)) ×
      Normal
        (product L1 (λl. real (1 - CDF p (real ∘ X l) t))) ×
      (1 -
```

```
      (1 -
       Normal
         (product L₂ (λl. real (1 - CDF p (real ∘ X l) t)))) ×
      (1 -
       Normal
         (product L₃ (λl. real (1 - CDF p (real ∘ X l) t))))) ×
     Normal
       (product L₄ (λl. real (1 - CDF p (real ∘ X l) t))) ×
     Normal
       (product L
          (λj.
               1 -
               real
                 (CDF p (real ∘ X (2 × j)) t ×
                  CDF p (real ∘ X (2 × j + 1)) t))))
```

[PROB_SEN_network_lem1]

```
 ⊢ ∀p L₁ L₂ L₃ L₄ L X t LL A J.
       FINITE L₁ ∧ L₁ ≠ {} ∧ FINITE L₂ ∧ L₂ ≠ {} ∧ FINITE L₃ ∧
       L₃ ≠ {} ∧ FINITE L₄ ∧ L₄ ≠ {} ∧ FINITE L ∧
       DISJOINT {0; 1; 3; 4} L ∧
       (∀i j.
            i ∈ L ∧ j ∈ L ∧ i ≠ j ⇒
            DISJOINT {2 × i; 2 × i + 1} {2 × j; 2 × j + 1}) ∧
       (∀i. i ∈ L ⇒ DISJOINT {2 × i; 2 × i + 1} {0; 1; 2; 3; 4}) ∧
       indep_sets p (λi. {X i}) (BIGUNION_o_BIGUNION LL A J) ∧
       disjoint_family_on
         (ind_set
            [{0}; L₁; L₂; L₃; L₄;
             {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L}])
         {0; 1; 2; 3; 4; 5} ∧ (J = {0; 1; 3; 4} ∪ L) ∧
       (A =
        (λj.
             if j ∈ L then {2 × j; 2 × j + 1}
             else ind_set [{0}; {1}; {}; {2; 3}; {4}] j)) ∧
       (LL =
        (λi.
             if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
               {i}
             else ind_set [{0}; L₁; L₂; L₃; L₄] i)) ⇒
       (prob p
          (DRBD_series
             (λj. DRBD_parallel (λa. DRBD_series X (LL a)) (A j))
             J) =
        prob p (X 0) ×
        Normal (product L₁ (λl. real (prob p (X l)))) ×
        (1 -
         (1 - Normal (product L₂ (λl. real (prob p (X l))))) ×
         (1 - Normal (product L₃ (λl. real (prob p (X l)))))) ×
```

```
        Normal (product L4 (λl. real (prob p (X l)))) ×
        Normal
          (product L
             (λj.
                  1 -
                  real
                    ((1 - prob p (X (2 × j))) ×
                     (1 - prob p (X (2 × j + 1)))))))
```

[PROB_SEN_network_lem2]

```
⊢ ∀p L1 L2 L3 L4 L X Y t.
       SEN_network_set_req p L1 L2 L3 L4 L
         (λi.
              if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
                {i}
              else ind_set [{0}; L1; L2; L3; L4] i)
         (λj.
              if j ∈ L then {2 × j; 2 × j + 1}
              else ind_set [{0}; {1}; {}; {2; 3}; {4}] j)
         ({0; 1; 3; 4} ∪ L)
         (event_set [(DRBD_event p Y t,0)] (rv_to_event p X t)) ⇒
       (prob p
          (DRBD_series
             (λj.
                  DRBD_parallel
                    (λa.
                         DRBD_series
                           (λi.
                                event_set [(DRBD_event p Y t,0)]
                                  (rv_to_event p X t) i)
                           ((λi.
                                 if
                                   i ∈
                                   {2 × i | i ∈ L} ∪
                                   {2 × i + 1 | i ∈ L}
                                 then
                                   {i}
                                 else
                                   ind_set [{0}; L1; L2; L3; L4]
                                     i) a))
                    ((λa.
                          if a ∈ L then {2 × a; 2 × a + 1}
                          else
                            ind_set [{0}; {1}; {}; {2; 3}; {4}]
                              a) j)) ({0; 1; 3; 4} ∪ L)) =
        prob p (DRBD_event p Y t) ×
        Normal
          (product L1 (λl. real (prob p (rv_to_event p X t l)))) ×
        (1 -
```

```
         (1 -
          Normal
            (product L2
               (λ l. real (prob p (rv_to_event p X t l))))) ×
         (1 -
          Normal
            (product L3
               (λ l. real (prob p (rv_to_event p X t l)))))) ×
        Normal
          (product L4 (λ l. real (prob p (rv_to_event p X t l)))) ×
        Normal
          (product L
             (λ j.
                  1 -
                  real
                    ((1 - prob p (rv_to_event p X t (2 × j))) ×
                     (1 - prob p (rv_to_event p X t (2 × j + 1)))))))
```

[PROB_SEN_plus_network_DFT]

```
 ⊢ ∀ p L1 L2 L3 L4 L X Y Ysa Ysd t.
        SEN_network_set_req p L1 L2 L3 L4 L
          (λ i.
               if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
                 {i}
               else ind_set [{0}; L1; L2; L3; L4] i)
          (λ j.
               if j ∈ L then {2 × j; 2 × j + 1}
               else ind_set [{0}; {1}; { }; {2; 3}; {4}] j)
          ({0; 1; 3; 4} ∪ L)
          (λ i.
               event_set [(DFT_event p (WSP Y Ysa Ysd) t,0)]
                 (rv_to_devent p X t) i) ∧
        (∀ i.
             i ∈
             L1 ∪ L2 ∪ L3 ∪ L4 ∪ {2 × i | i ∈ L} ∪
             {2 × i + 1 | i ∈ L} ⇒
             rv_gt0_ninfinity [X i]) ⇒
        (prob p
           (DFT_event p
              (n_OR
                 (MAP
                    (λ i.
                         if i = 0 then WSP Y Ysa Ysd
                         else if i = 1 then
                           n_OR (MAP X (SET_TO_LIST L1))
                         else if i = 3 then
                           D_AND (n_OR (MAP X (SET_TO_LIST L2)))
                             (n_OR (MAP X (SET_TO_LIST L3)))
                         else if i = 4 then
```

```
                    n_OR (MAP X (SET_TO_LIST L4))
                  else D_AND (X (2 × i)) (X (2 × i + 1)))
              (SET_TO_LIST ({0; 1; 3; 4} ∪ L)))) t) =
   1 -
   (1 - prob p (DFT_event p (WSP Y Ysa Ysd) t)) ×
   Normal
     (product L1 (λ l. real (1 - CDF p (real ∘ X l) t))) ×
   (1 -
    (1 -
     Normal
       (product L2 (λ l. real (1 - CDF p (real ∘ X l) t)))) ×
    (1 -
     Normal
       (product L3 (λ l. real (1 - CDF p (real ∘ X l) t))))) ×
   Normal
     (product L4 (λ l. real (1 - CDF p (real ∘ X l) t))) ×
   Normal
     (product L
        (λ j.
           1 -
           real
             (CDF p (real ∘ X (2 × j)) t ×
              CDF p (real ∘ X (2 × j + 1)) t))))
```

[PROB_SEN_plus_network_DFT_spares]

```
⊢ ∀ p L1 L2 L3 L4 L X Y Ysa Ysd t.
    SEN_network_set_req p L1 L2 L3 L4 L
      (λ i.
         if i ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
           {i}
         else ind_set [{0}; L1; L2; L3; L4] i)
      (λ j.
         if j ∈ L then {2 × j; 2 × j + 1}
         else ind_set [{0}; {1}; { }; {2; 3}; {4}] j)
      ({0; 1; 3; 4} ∪ L)
      (λ i.
         event_set
           [(DFT_event p (WSP (Y 0) (Ysa 0) (Ysd 0)) t,0)]
           (rv_to_devent p X t) i) ∧
    (∀ i.
       i ∈
       L1 ∪ L2 ∪ L3 ∪ L4 ∪ {2 × i | i ∈ L} ∪
       {2 × i + 1 | i ∈ L} ⇒
       rv_gt0_ninfinity [X i]) ∧
    (∀ i. i ∈ L1 ⇒ (X i = WSP (Y i) (Ysa i) (Ysd i))) ⇒
    (prob p
       (DFT_event p
          (n_OR
             (MAP
```

```
                (λ i.
                     if i = 0 then WSP (Y 0) (Ysa 0) (Ysd 0)
                     else if i = 1 then
                       n_OR (MAP X (SET_TO_LIST L1))
                     else if i = 3 then
                       D_AND (n_OR (MAP X (SET_TO_LIST L2)))
                         (n_OR (MAP X (SET_TO_LIST L3)))
                     else if i = 4 then
                       n_OR (MAP X (SET_TO_LIST L4))
                     else D_AND (X (2 × i)) (X (2 × i + 1)))
                (SET_TO_LIST ({0; 1; 3; 4} ∪ L)))) t) =
      1 -
      Normal
        (product ({0} ∪ L1)
           (λ l.
                real
                  (1 -
                   prob p
                     (DFT_event p (WSP (Y l) (Ysa l) (Ysd l))
                        t)))) ×
      (1 -
       (1 -
        Normal
          (product L2 (λ l. real (1 - CDF p (real ∘ X l) t)))) ×
       (1 -
        Normal
          (product L3 (λ l. real (1 - CDF p (real ∘ X l) t))))) ×
      Normal
        (product L4 (λ l. real (1 - CDF p (real ∘ X l) t))) ×
      Normal
        (product L
           (λ j.
                1 -
                real
                  (CDF p (real ∘ X (2 × j)) t ×
                   CDF p (real ∘ X (2 × j + 1)) t))))
```

[PROB_sSEN_DFT]

```
 ⊢ ∀ p X Y Ysa Ysd t L.
      DISJOINT {0} L ∧ FINITE L ∧ L ≠ { } ∧
      indep_sets p
        (λ i.
             {rv_to_devent p
                (λ i. if i = 0 then WSP Y Ysa Ysd else X i) t i})
        ({0} ∪ L) ⇒
      (prob p
         (DFT_event p
            (n_OR
               (MAP (λ i. if i = 0 then WSP Y Ysa Ysd else X i)
```

```
            (SET_TO_LIST ({0} ∪ L)))) t) =
    1 -
    (1 - prob p (DFT_event p (WSP Y Ysa Ysd) t)) ×
    Normal
      (product L
         (λi. real (1 - prob p (DFT_event p (X i) t)))))
```

[PROB_sSEN_DFT_CDF]

```
⊢ ∀p X Y Ysa Ysd t L.
    DISJOINT {0} L ∧ FINITE L ∧ L ≠ {} ∧
    indep_sets p
      (λi.
         {event_set [(DFT_event p (WSP Y Ysa Ysd) t,0)]
            (rv_to_devent p X t) i}) ({0} ∪ L) ∧
    (∀i. i ∈ L ⇒ rv_gt0_ninfinity [X i]) ⇒
    (prob p
       (DFT_event p
          (n_OR
             (MAP (λi. if i = 0 then WSP Y Ysa Ysd else X i)
                (SET_TO_LIST ({0} ∪ L)))) t) =
     1 -
     (1 - prob p (DFT_event p (WSP Y Ysa Ysd) t)) ×
     Normal (product L (λi. real (1 - CDF p (real ∘ X i) t))))
```

[PROB_sSEN_DFT_CDF_lem]

```
⊢ ∀p X Y Ysa Ysd t L.
    DISJOINT {0} L ∧ FINITE L ∧ L ≠ {} ∧
    indep_sets p
      (λi.
         {rv_to_devent p
            (λi. if i = 0 then WSP Y Ysa Ysd else X i) t i})
      ({0} ∪ L) ∧ (∀i. i ∈ L ⇒ rv_gt0_ninfinity [X i]) ⇒
    (prob p
       (DFT_event p
          (n_OR
             (MAP (λi. if i = 0 then WSP Y Ysa Ysd else X i)
                (SET_TO_LIST ({0} ∪ L)))) t) =
     1 -
     (1 - prob p (DFT_event p (WSP Y Ysa Ysd) t)) ×
     Normal (product L (λi. real (1 - CDF p (real ∘ X i) t))))
```

[PROB_sSEN_DFT_spares]

```
⊢ ∀p X Y Ysa Ysd t L1 L2.
    DISJOINT L1 L2 ∧ FINITE L1 ∧ L1 ≠ {} ∧ FINITE L2 ∧
    L2 ≠ {} ∧ (∀i. i ∈ L2 ⇒ rv_gt0_ninfinity [X i]) ∧
    indep_sets p
      (λi.
         {rv_to_devent p
```

```
              (λ i.
                   if i ∈ L1 then WSP (Y i) (Ysa i) (Ysd i)
                   else X i) t i}) (L1 ∪ L2) ⇒
      (prob p
         (DFT_event p
            (n_OR
               (MAP
                  (λ i.
                       if i ∈ L1 then WSP (Y i) (Ysa i) (Ysd i)
                       else X i) (SET_TO_LIST (L1 ∪ L2)))) t) =
       1 -
       Normal
         (product L1
            (λ i.
                 real
                   (1 -
                    prob p
                      (DFT_event p (WSP (Y i) (Ysa i) (Ysd i))
                         t)))) ×
       Normal
         (product L2 (λ i. real (1 - CDF p (real ∘ X i) t))))
```

[Q_dSEN_lem3]

```
⊢ ∀ p X Y Ysa Ysd t L1 L2 L3.
      disjoint_family_on (ind_set [{0}; L1; L2; L3])
        {0; 1; 2; 3} ∧ FINITE L1 ∧ FINITE L2 ∧ FINITE L3 ∧
      0 ≤ t ⇒
      (DFT_event p
         (n_OR
            (MAP
               (λ j.
                    n_AND
                      (MAP
                         (λ i.
                              n_OR
                                (MAP
                                   (λ i.
                                        if i = 0 then
                                          WSP Y Ysa Ysd
                                        else X i)
                                   (SET_TO_LIST
                                      (ind_set
                                         [{0}; L1; L2; L3] i))))
                         (SET_TO_LIST
                            (ind_set [{0}; {1; 2}; {3}] j))))
               (SET_TO_LIST {0; 1; 2}))) t =
       DFT_event p
         (n_OR
            (MAP
```

```
              (λ i.
                   if i = 0 then WSP Y Ysa Ysd
                   else if i = 1 then
                     D_AND (n_OR (MAP X (SET_TO_LIST L1)))
                       (n_OR (MAP X (SET_TO_LIST L2)))
                   else n_OR (MAP X (SET_TO_LIST L3)))
              (SET_TO_LIST {0; 1; 2}))) t)
```

[Q_dSEN_network_plus]

```
 ⊢ ∀p L1 L2 L3 L4 L X Y Ysa Ysd t.
       FINITE L1 ∧ L1 ≠ { } ∧ FINITE L2 ∧ L2 ≠ { } ∧ FINITE L3 ∧
       L3 ≠ { } ∧ FINITE L4 ∧ L4 ≠ { } ∧ FINITE L ∧
       DISJOINT {0; 1; 3; 4} L ∧
       (∀i. i ∈ L ⇒ DISJOINT {2 × i; 2 × i + 1} {0; 1; 2; 3; 4}) ∧
       disjoint_family_on
         (ind_set
            [{0}; L1; L2; L3; L4;
             {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L}])
         {0; 1; 2; 3; 4; 5} ⇒
       (DFT_event p
          (n_OR
             (MAP
                (λ i.
                     if i = 0 then WSP Y Ysa Ysd
                     else if i = 1 then
                       n_OR (MAP X (SET_TO_LIST L1))
                     else if i = 3 then
                       D_AND (n_OR (MAP X (SET_TO_LIST L2)))
                         (n_OR (MAP X (SET_TO_LIST L3)))
                     else if i = 4 then
                       n_OR (MAP X (SET_TO_LIST L4))
                     else D_AND (X (2 × i)) (X (2 × i + 1)))
                (SET_TO_LIST ({0; 1; 3; 4} ∪ L)))) t =
        BIGUNION
          {BIGINTER
            {BIGUNION
              {event_set [(DFT_event p (WSP Y Ysa Ysd) t,0)]
                 (rv_to_devent p X t) i |
               i ∈
               if a ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
                 {a} else ind_set [{0}; L1; L2; L3; L4] a} |
             a |
             a ∈ if j ∈ L then {2 × j; 2 × j + 1}
             else ind_set [{0}; {1}; { }; {2; 3}; {4}] j} |
           j |
           j ∈ {0; 1; 3; 4} ∪ L})
```

[Q_dSEN_network_plus_lem1]

```
 ⊢ ∀L1 L2 L3 L4 L X Y.
       FINITE L1 ∧ FINITE L2 ∧ FINITE L3 ∧ FINITE L4 ∧ FINITE L ⇒
```

```
      (n_OR
         (MAP
            (λ i.
                 if i = 0 then Y
                 else if i = 1 then
                   n_OR (MAP X (SET_TO_LIST L1))
                 else if i = 3 then
                   D_AND (n_OR (MAP X (SET_TO_LIST L2)))
                     (n_OR (MAP X (SET_TO_LIST L3)))
                 else if i = 4 then
                   n_OR (MAP X (SET_TO_LIST L4))
                 else D_AND (X (2 × i)) (X (2 × i + 1)))
            (SET_TO_LIST ({0; 1; 3; 4} ∪ L))) =
       nR_AND
         (λ i.
              if i = 0 then Y
              else if i = 1 then nR_AND X L1
              else if i = 3 then
                R_OR (nR_AND X L2) (nR_AND X L3)
              else if i = 4 then nR_AND X L4
              else R_OR (X (2 × i)) (X (2 × i + 1)))
         ({0; 1; 3; 4} ∪ L))
```

[Q_dSEN_network_plus_lem2]

```
 ⊢ ∀ p L1 L2 L3 L4 L X Y t.
      FINITE L1 ∧ L1 ≠ { } ∧ FINITE L2 ∧ L2 ≠ { } ∧ FINITE L3 ∧
      L3 ≠ { } ∧ FINITE L4 ∧ L4 ≠ { } ∧ FINITE L ∧
      DISJOINT {0; 1; 3; 4} L ∧
      (∀ i. i ∈ L ⇒ DISJOINT {2 × i; 2 × i + 1} {0; 1; 2; 3; 4}) ∧
      disjoint_family_on
        (ind_set
           [{0}; L1; L2; L3; L4;
            {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L}])
        {0; 1; 2; 3; 4; 5} ⇒
      (DFT_event p
         (n_OR
            (MAP
               (λ i.
                    if i = 0 then Y
                    else if i = 1 then
                      n_OR (MAP X (SET_TO_LIST L1))
                    else if i = 3 then
                      D_AND (n_OR (MAP X (SET_TO_LIST L2)))
                        (n_OR (MAP X (SET_TO_LIST L3)))
                    else if i = 4 then
                      n_OR (MAP X (SET_TO_LIST L4))
                    else D_AND (X (2 × i)) (X (2 × i + 1)))
               (SET_TO_LIST ({0; 1; 3; 4} ∪ L)))) t =
       BIGUNION
```

```
    {BIGINTER
      {BIGUNION
        {event_set [(DFT_event p Y t,0)]
          (rv_to_devent p X t) i |
        i ∈
        if a ∈ {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L} then
          {a} else ind_set [{0}; L1; L2; L3; L4] a} |
      a |
      a ∈ if j ∈ L then {2 × j; 2 × j + 1}
      else ind_set [{0}; {1}; {}; {2; 3}; {4}] j} |
    j |
    j ∈ {0; 1; 3; 4} ∪ L})
```

[Q_dSEN_plus_lem1]

```
⊢ ∀L1 L2 L3 X Y.
     FINITE L1 ∧ FINITE L2 ∧ FINITE L3 ⇒
     (n_OR
        (MAP
           (λi.
                if i = 0 then Y
                else if i = 1 then
                  D_AND (n_OR (MAP X (SET_TO_LIST L1)))
                    (n_OR (MAP X (SET_TO_LIST L2)))
                else n_OR (MAP X (SET_TO_LIST L3)))
           (SET_TO_LIST {0; 1; 2})) =
      nR_AND
        (λi.
             if i = 0 then Y
             else if i = 1 then
               R_OR (nR_AND X L1) (nR_AND X L2)
             else nR_AND X L3) {0; 1; 2})
```

[Q_dSEN_plus_lem2]

```
⊢ ∀p L1 L2 L3 X Y t.
     disjoint_family_on (ind_set [{0}; L1; L2; L3])
       {0; 1; 2; 3} ∧ FINITE L1 ∧ FINITE L2 ∧ FINITE L3 ∧
     0 ≤ t ⇒
     (DFT_event p
        (n_OR
           (MAP
              (λi.
                   if i = 0 then Y
                   else if i = 1 then
                     D_AND (n_OR (MAP X (SET_TO_LIST L1)))
                       (n_OR (MAP X (SET_TO_LIST L2)))
                   else n_OR (MAP X (SET_TO_LIST L3)))
              (SET_TO_LIST {0; 1; 2}))) t =
      BIGUNION
        {BIGINTER
```

```
{BIGUNION
  {event_set [(DFT_event p Y t,0)]
    (rv_to_devent p X t) i |
  i ∈ ind_set [{0}; L1; L2; L3] a} |
 a ∈ ind_set [{0}; {1; 2}; {3}] j} |
j ∈ {0; 1; 2}})
```

[Q_SEN_network_nR_AND_DRBD_series]

```
⊢ ∀p L1 L2 L3 L4 L X t.
      FINITE L1 ∧ L1 ≠ {} ∧ FINITE L2 ∧ L2 ≠ {} ∧ FINITE L3 ∧
      L3 ≠ {} ∧ FINITE L4 ∧ L4 ≠ {} ∧ FINITE L ∧
      DISJOINT {0; 1; 3; 4} L ∧
      (∀i. i ∈ L ⇒ DISJOINT {2 × i; 2 × i + 1} {0; 1; 2; 3; 4}) ⇒
      (DRBD_event p
         (nR_AND
            (λi.
                 if i = 0 then X 0
                 else if i = 1 then nR_AND X L1
                 else if i = 3 then
                   R_OR (nR_AND X L2) (nR_AND X L3)
                 else if i = 4 then nR_AND X L4
                 else R_OR (X (2 × i)) (X (2 × i + 1)))
            ({0; 1; 3; 4} ∪ L)) t =
       DRBD_series
         (λj.
              DRBD_parallel
                (λa.
                     DRBD_series (λi. rv_to_event p X t i)
                       ((λa.
                             if a = 0 then {0}
                             else if a = 1 then L1
                             else if a = 2 then L2
                             else if a = 3 then L3
                             else if a = 4 then L4
                             else {a}) a))
                ((λj.
                      if j = 0 then {0}
                      else if j = 1 then {1}
                      else if j ∈ L then {2 × j; 2 × j + 1}
                      else if j = 3 then {2; 3}
                      else {4}) j)) ({0; 1; 3; 4} ∪ L))
```

[Q_SEN_network_nR_AND_DRBD_series_lem]

```
⊢ ∀p L1 L2 L3 L4 L X Y t.
      FINITE L1 ∧ L1 ≠ {} ∧ FINITE L2 ∧ L2 ≠ {} ∧ FINITE L3 ∧
      L3 ≠ {} ∧ FINITE L4 ∧ L4 ≠ {} ∧ FINITE L ∧
      DISJOINT {0; 1; 3; 4} L ∧
      (∀i. i ∈ L ⇒ DISJOINT {2 × i; 2 × i + 1} {0; 1; 2; 3; 4}) ∧
      disjoint_family_on
```

```
      (ind_set
         [{0}; L1; L2; L3; L4;
          {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L}])
      {0; 1; 2; 3; 4; 5} ⇒
    (DRBD_event p
       (nR_AND
          (λ i.
               if i = 0 then Y
               else if i = 1 then nR_AND X L1
               else if i = 3 then
                 R_OR (nR_AND X L2) (nR_AND X L3)
               else if i = 4 then nR_AND X L4
               else R_OR (X (2 × i)) (X (2 × i + 1)))
          ({0; 1; 3; 4} ∪ L)) t =
     DRBD_series
       (λ j.
            DRBD_parallel
              (λ a.
                   DRBD_series
                     (λ i.
                          event_set [(DRBD_event p Y t,0)]
                            (rv_to_event p X t) i)
                     ((λ i.
                           if
                             i ∈
                             {2 × i | i ∈ L} ∪
                             {2 × i + 1 | i ∈ L}
                           then
                             {i}
                           else
                             ind_set [{0}; L1; L2; L3; L4] i)
                        a))
              ((λ j.
                    if j ∈ L then {2 × j; 2 × j + 1}
                    else
                      ind_set [{0}; {1}; {}; {2; 3}; {4}] j)
                 j)) ({0; 1; 3; 4} ∪ L))
```

[Q_SEN_network_nR_AND_DRBD_series_one_spare]

```
 ⊢ ∀ p L1 L2 L3 L4 L X Y Ysa Ysd t.
     FINITE L1 ∧ L1 ≠ {} ∧ FINITE L2 ∧ L2 ≠ {} ∧ FINITE L3 ∧
     L3 ≠ {} ∧ FINITE L4 ∧ L4 ≠ {} ∧ FINITE L ∧
     DISJOINT {0; 1; 3; 4} L ∧
     (∀ i. i ∈ L ⇒ DISJOINT {2 × i; 2 × i + 1} {0; 1; 2; 3; 4}) ∧
     disjoint_family_on
       (ind_set
          [{0}; L1; L2; L3; L4;
           {2 × i | i ∈ L} ∪ {2 × i + 1 | i ∈ L}])
       {0; 1; 2; 3; 4; 5} ⇒
```

```
      (DRBD_event p
         (nR_AND
            (λ i.
                 if i = 0 then R_WSP Y Ysa Ysd
                 else if i = 1 then nR_AND X L1
                 else if i = 3 then
                   R_OR (nR_AND X L2) (nR_AND X L3)
                 else if i = 4 then nR_AND X L4
                 else R_OR (X (2 × i)) (X (2 × i + 1)))
            ({0; 1; 3; 4} ∪ L)) t =
       DRBD_series
         (λ j.
              DRBD_parallel
                (λ a.
                     DRBD_series
                       (λ i.
                            event_set
                              [(DRBD_event p (R_WSP Y Ysa Ysd)
                                  t,0)] (rv_to_event p X t) i)
                       ((λ i.
                             if
                               i ∈
                               {2 × i | i ∈ L} ∪
                               {2 × i + 1 | i ∈ L}
                             then
                               {i}
                             else
                               ind_set [{0}; L1; L2; L3; L4] i)
                          a))
                ((λ j.
                      if j ∈ L then {2 × j; 2 × j + 1}
                      else
                        ind_set [{0}; {1}; {}; {2; 3}; {4}] j)
                   j)) ({0; 1; 3; 4} ∪ L))
```

[real_mul_real]

```
 ⊢ ∀ a b.
      a ≠ PosInf ∧ a ≠ NegInf ∧ b ≠ PosInf ∧ b ≠ NegInf ⇒
      (real a × real b = real (a × b))
```

[Rel_DRBD_SEN_plus]

```
 ⊢ ∀ p X Y Ys_a Ys_d Z Zs_a Zs_d t L1 L2.
      SEN_set_req p L1 L2 (ind_set [{0}; L1; L2; {3}])
        (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
        (event_set
           [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0);
            (DRBD_event p (R_WSP Z Zs_a Zs_d) t,3)]
           (rv_to_event p X t)) ⇒
      (prob p
```

```
          (DRBD_event p
             (nR_AND
                (λ i.
                      if i = 0 then R_WSP Y Ys_a Ys_d
                      else if i = 1 then
                        R_OR (nR_AND X L₁) (nR_AND X L₂)
                      else R_WSP Z Zs_a Zs_d) {0; 1; 2}) t) =
       Rel p (R_WSP Y Ys_a Ys_d) t ×
       Rel p (R_WSP Z Zs_a Zs_d) t ×
       (1 -
        (1 - Normal (product L₁ (λ l. real (Rel p (X l) t)))) ×
        (1 - Normal (product L₂ (λ l. real (Rel p (X l) t))))))
```

[Rel_DRBD_SEN_plus1]

```
 ⊢ ∀ p X Y Ys_a Ys_d Z Zs_a Zs_d t L₁ L₂ f_y f_z f_condY f_condZ
        f_ysy f_zsz.
        SEN_set_req p L₁ L₂ (ind_set [{0}; L₁; L₂; {3}])
          (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
          (event_set
             [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0);
              (DRBD_event p (R_WSP Z Zs_a Zs_d) t,3)]
             (rv_to_event p X t)) ∧ prob_space p ∧
        (∀ s.
             ALL_DISTINCT
               [Ys_a s; Ys_d s; Y s; Zs_a s; Zs_d s; Z s]) ∧
        DISJOINT_WSP Y Ys_a Ys_d t ∧ DISJOINT_WSP Z Zs_a Zs_d t ∧
        rv_gt0_ninfinity [Ys_a; Ys_d; Y; Zs_a; Zs_d; Z] ∧ 0 ≤ t ∧
        (∀ y.
             cond_density lborel lborel p (real ∘ Ys_a)
               (real ∘ Y) y f_ysy f_y f_condY) ∧
        den_gt0_ninfinity f_ysy f_y f_condY ∧
        indep_var p lborel (real ∘ Ys_d) lborel (real ∘ Y) ∧
        cont_CDF p (real ∘ Ys_d) ∧
        measurable_CDF p (real ∘ Ys_d) ∧
        (∀ z.
             cond_density lborel lborel p (real ∘ Zs_a)
               (real ∘ Z) z f_zsz f_z f_condZ) ∧
        den_gt0_ninfinity f_zsz f_z f_condZ ∧
        indep_var p lborel (real ∘ Zs_d) lborel (real ∘ Z) ∧
        cont_CDF p (real ∘ Zs_d) ∧ measurable_CDF p (real ∘ Zs_d) ⇒
        (prob p
           (DRBD_event p
              (nR_AND
                 (λ i.
                       if i = 0 then R_WSP Y Ys_a Ys_d
                       else if i = 1 then
                         R_OR (nR_AND X L₁) (nR_AND X L₂)
                       else R_WSP Z Zs_a Zs_d) {0; 1; 2}) t) =
         (1 -
```

```
      (pos_fn_integral lborel
         (λ y.
              indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_y y ×
              pos_fn_integral lborel
                (λ x.
                     indicator_fn {w | y < w ∧ w ≤ t} x ×
                     f_condY y x)) +
       pos_fn_integral lborel
         (λ y.
              f_y y ×
              (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
               CDF p (real ∘ Ys_d) y)))) ×
     (1 -
      (pos_fn_integral lborel
         (λ y.
              indicator_fn {u | 0 ≤ u ∧ u ≤ t} y × f_z y ×
              pos_fn_integral lborel
                (λ x.
                     indicator_fn {w | y < w ∧ w ≤ t} x ×
                     f_condZ y x)) +
       pos_fn_integral lborel
         (λ y.
              f_z y ×
              (indicator_fn {u | 0 ≤ u ∧ u ≤ t} y ×
               CDF p (real ∘ Zs_d) y)))) ×
     (1 -
      (1 - Normal (product L1 (λ l. real (Rel p (X l) t)))) ×
      (1 - Normal (product L2 (λ l. real (Rel p (X l) t)))))
```

[Rel_DRBD_SEN_plus_broadcast]

```
 ⊢ ∀ p X Y Ys_a Ys_d t L1 L2 L3.
       SEN_broad_set_req p L1 L2 L3 (ind_set [{0}; L1; L2; L3])
         (ind_set [{0}; {1; 2}; {3}]) {0; 1; 2}
         (event_set [(DRBD_event p (R_WSP Y Ys_a Ys_d) t,0)]
            (rv_to_event p X t)) ⇒
       (prob p
          (DRBD_event p
             (nR_AND
                (λ i.
                     if i = 0 then R_WSP Y Ys_a Ys_d
                     else if i = 1 then
                       R_OR (nR_AND X L1) (nR_AND X L2)
                     else nR_AND X L3) {0; 1; 2}) t) =
        Rel p (R_WSP Y Ys_a Ys_d) t ×
        Normal (product L3 (λ l. real (Rel p (X l) t))) ×
        (1 -
         (1 - Normal (product L1 (λ l. real (Rel p (X l) t)))) ×
         (1 - Normal (product L2 (λ l. real (Rel p (X l) t)))))
```

[Rel_sSEN]

```
⊢ ∀p X Y Ysa Ysd t L.
     DISJOINT {0} L ∧ FINITE L ∧ L ≠ { } ∧
     indep_sets p
       (λi.
            {event_set [(DRBD_event p (R_WSP Y Ysa Ysd) t,0)]
               (rv_to_event p X t) i}) ({0} ∪ L) ⇒
     (prob p
        (DRBD_event p
           (nR_AND
              (λi. if i = 0 then R_WSP Y Ysa Ysd else X i)
              ({0} ∪ L)) t) =
      Rel p (R_WSP Y Ysa Ysd) t ×
      Normal (product L (λl. real (Rel p (X l) t))))
```

[Rel_sSEN_spares]

```
⊢ ∀p X Y Ysa Ysd t L1 L2.
     DISJOINT L1 L2 ∧ FINITE L1 ∧ L1 ≠ { } ∧ FINITE L2 ∧
     L2 ≠ { } ∧
     indep_sets p
       (λi.
            {if i ∈ L1 then
               DRBD_event p (R_WSP (Y i) (Ysa i) (Ysd i)) t
             else rv_to_event p X t i}) (L1 ∪ L2) ⇒
     (prob p
        (DRBD_event p
           (nR_AND
              (λi.
                   if i ∈ L1 then R_WSP (Y i) (Ysa i) (Ysd i)
                   else X i) (L1 ∪ L2)) t) =
      Normal
        (product L1
           (λi. real (Rel p (R_WSP (Y i) (Ysa i) (Ysd i)) t))) ×
      Normal (product L2 (λi. real (Rel p (X i) t))))
```

[SEN_DRBD_DFT_broadcast_eq]

```
⊢ ∀L1 L2 L3 Y Ysa Ysd X.
     FINITE L1 ∧ FINITE L2 ∧ FINITE L3 ∧
     (∀s. ALL_DISTINCT [Y s; Ysa s; Ysd s]) ⇒
     (nR_AND
        (λi.
             if i = 0 then R_WSP Y Ysa Ysd
             else if i = 1 then
               R_OR (nR_AND X L1) (nR_AND X L2)
             else nR_AND X L3) {0; 1; 2} =
      n_OR
        (MAP
           (λi.
```

```
                 if i = 0 then WSP Y Ysa Ysd
                 else if i = 1 then
                   D_AND (n_OR (MAP X (SET_TO_LIST L1)))
                     (n_OR (MAP X (SET_TO_LIST L2)))
                 else n_OR (MAP X (SET_TO_LIST L3)))
            (SET_TO_LIST {0; 1; 2})))
```

[SEN_DRBD_DFT_broadcast_event_eq]

```
 ⊢ ∀p L1 L2 L3 X Y Ysa Ysd t.
      FINITE L1 ∧ FINITE L2 ∧ FINITE L3 ∧
      (∀s. ALL_DISTINCT [Y s; Ysa s; Ysd s]) ⇒
      (DRBD_event p
         (nR_AND
            (λi.
                 if i = 0 then R_WSP Y Ysa Ysd
                 else if i = 1 then
                   R_OR (nR_AND X L1) (nR_AND X L2)
                 else nR_AND X L3) {0; 1; 2}) t =
       p_space p DIFF
       DFT_event p
         (n_OR
            (MAP
               (λi.
                    if i = 0 then WSP Y Ysa Ysd
                    else if i = 1 then
                      D_AND (n_OR (MAP X (SET_TO_LIST L1)))
                        (n_OR (MAP X (SET_TO_LIST L2)))
                    else n_OR (MAP X (SET_TO_LIST L3)))
               (SET_TO_LIST {0; 1; 2}))) t)
```

[SEN_DRBD_DFT_terminal_eq]

```
 ⊢ ∀L1 L2 X Y Ysa Ysd Z Zsa Zsd.
      FINITE L1 ∧ FINITE L2 ∧
      (∀s. ALL_DISTINCT [Y s; Ysa s; Ysd s; Z s; Zsa s; Zsd s]) ⇒
      (nR_AND
         (λi.
              if i = 0 then R_WSP Y Ysa Ysd
              else if i = 1 then
                R_OR (nR_AND X L1) (nR_AND X L2)
              else R_WSP Z Zsa Zsd) {0; 1; 2} =
       n_OR
         (MAP
            (λi.
                 if i = 0 then WSP Y Ysa Ysd
                 else if i = 1 then
                   D_AND (n_OR (MAP X (SET_TO_LIST L1)))
                     (n_OR (MAP X (SET_TO_LIST L2)))
                 else WSP Z Zsa Zsd) (SET_TO_LIST {0; 1; 2})))
```

[SEN_DRBD_DFT_terminal_event_eq]

```
⊢ ∀p L1 L2 X Y Ysa Ysd Z Zsa Zsd t.
      FINITE L1 ∧ FINITE L2 ∧
      (∀s. ALL_DISTINCT [Y s; Ysa s; Ysd s; Z s; Zsa s; Zsd s]) ⇒
      (DRBD_event p
         (nR_AND
            (λi.
                 if i = 0 then R_WSP Y Ysa Ysd
                 else if i = 1 then
                   R_OR (nR_AND X L1) (nR_AND X L2)
                 else R_WSP Z Zsa Zsd) {0; 1; 2}) t =
       p_space p DIFF
       DFT_event p
         (n_OR
            (MAP
               (λi.
                    if i = 0 then WSP Y Ysa Ysd
                    else if i = 1 then
                      D_AND (n_OR (MAP X (SET_TO_LIST L1)))
                        (n_OR (MAP X (SET_TO_LIST L2)))
                    else WSP Z Zsa Zsd) (SET_TO_LIST {0; 1; 2})))
         t)
```

[SEN_nR_AND]

```
⊢ ∀p X Y Ys_a Ys_d Z Zs_a Zs_d t L1 L2.
      DISJOINT3 {0; 3} L1 L2 ∧ FINITE L1 ∧ FINITE L2 ∧
      L1 ≠ { } ∧ L2 ≠ { } ⇒
      (DRBD_event p
         (nR_AND
            (λi.
                 if i = 0 then R_WSP Y Ys_a Ys_d
                 else if i = 1 then
                   R_OR (nR_AND X L1) (nR_AND X L2)
                 else R_WSP Z Zs_a Zs_d) {0; 1; 2}) t =
       DRBD_series
         (λj.
              DRBD_parallel
                (λa.
                     DRBD_series
                       (λi.
                            event_set
                              [(DRBD_event p
                                  (R_WSP Y Ys_a Ys_d) t,0);
                               (DRBD_event p
                                  (R_WSP Z Zs_a Zs_d) t,3)]
                              (rv_to_event p X t) i)
                       (ind_set [{0}; L1; L2; {3}] a))
                (ind_set [{0}; {1; 2}; {3}] j)) {0; 1; 2})
```

[SEN_plus_broadcast_nR_AND]

```
⊢ ∀p X Y Ys_a Ys_d t L1 L2 L3.
      disjoint_family_on (ind_set [{0}; L1; L2; L3])
        {0; 1; 2; 3} ∧ FINITE L1 ∧ FINITE L2 ∧ FINITE L3 ∧
      L1 ≠ {} ∧ L2 ≠ {} ∧ L3 ≠ {} ⇒
      (DRBD_event p
         (nR_AND
            (λi.
                 if i = 0 then R_WSP Y Ys_a Ys_d
                 else if i = 1 then
                   R_OR (nR_AND X L1) (nR_AND X L2)
                 else nR_AND X L3) {0; 1; 2}) t =
       DRBD_series
         (λj.
              DRBD_parallel
                (λa.
                     DRBD_series
                       (λi.
                            event_set
                              [(DRBD_event p
                                  (R_WSP Y Ys_a Ys_d) t,0)]
                              (rv_to_event p X t) i)
                       (ind_set [{0}; L1; L2; L3] a))
                (ind_set [{0}; {1; 2}; {3}] j)) {0; 1; 2})
```

[sSEN_DRBD_DFT_eq]

```
⊢ ∀L Y Ysa Ysd X.
      FINITE L ∧ (∀s. ALL_DISTINCT [Y s; Ysa s; Ysd s]) ⇒
      (nR_AND (λi. if i = 0 then R_WSP Y Ysa Ysd else X i)
         ({0} ∪ L) =
       n_OR
         (MAP (λi. if i = 0 then WSP Y Ysa Ysd else X i)
            (SET_TO_LIST ({0} ∪ L))))
```

[sSEN_DRBD_DFT_event_eq]

```
⊢ ∀p L Y Ysa Ysd X t.
      FINITE L ∧ (∀s. ALL_DISTINCT [Y s; Ysa s; Ysd s]) ⇒
      (DRBD_event p
         (nR_AND (λi. if i = 0 then R_WSP Y Ysa Ysd else X i)
            ({0} ∪ L)) t =
       p_space p DIFF
       DFT_event p
         (n_OR
            (MAP (λi. if i = 0 then WSP Y Ysa Ysd else X i)
               (SET_TO_LIST ({0} ∪ L))) t))
```

[sSEN_nR_AND]

```
⊢ ∀p X Y Ysa Ysd t L.
      DISJOINT {0} L ∧ FINITE L ∧ L ≠ {} ⇒
      (DRBD_event p
         (nR_AND (λi. if i = 0 then R_WSP Y Ysa Ysd else X i)
            ({0} ∪ L)) t =
       DRBD_series
         (λi.
              event_set [(DRBD_event p (R_WSP Y Ysa Ysd) t,0)]
                (rv_to_event p X t) i) ({0} ∪ L))
```

[sSEN_nR_AND_spares]

```
⊢ ∀p X Y Ysa Ysd t L1 L2.
      DISJOINT L1 L2 ∧ FINITE L1 ∧ L1 ≠ {} ∧ FINITE L2 ∧
      L2 ≠ {} ⇒
      (DRBD_event p
         (nR_AND
            (λi.
                 if i ∈ L1 then R_WSP (Y i) (Ysa i) (Ysd i)
                 else X i) (L1 ∪ L2)) t =
       DRBD_series
         (λi.
              if i ∈ L1 then
                DRBD_event p (R_WSP (Y i) (Ysa i) (Ysd i)) t
              else rv_to_event p X t i) (L1 ∪ L2))
```